1. Introduction

This paper attempts at fixing some guide-posts on the relation between variety, consumption and growth in supply-led and demand-constrained models of economic growth. For the sake of simplicity, the former will be identified with a class of general-equilibrium models with R&D, the latter will be obtained from the former with a modicum of crucial modifications, thus preserving a certain controlled comparability between the two theoretical frames.

In extreme synthesis, the economic literature identifies at least three different ways in which variety may affect the pattern of consumption.

The first case occurs with the introduction of radically-new goods responding to previously unmet needs. These goods convey new service characteristics, or at least a combination of previously unavailable characteristics. For instance, the creation of the internal-combustion-engine powered automobile offered a new mix of transportation services combining speed with flexibility of use in time and space and lack of animal-waste. Such a vector of service characteristics could not be supplied by the competing land-transportation-systems of the time based on trains and horses (Bresnahan and Gordon, 1997). A similar case is offered by the first introduction of domestic refrigerators bringing to previously un-imaginable levels the time-flexibility of fresh-food consumption. Holding to Becker's (1965) and Lancaster's (1971) models of a *fixed set of service characteristics* supplied in different degree and composition by the home-production of consumption services using home-labour and goods as inputs, Bresnahan and Gordon (1997) suggest that the innovation examples just given correspond to the creation of new inputs for consumption-service production, which enable this production process to meet 'objective (previously) unmet needs' (*ibidem*, p.11). Other authors object that there is a process of learning and preference-formation associated with the creation of new goods which is not fully consistent with Becker's and Lancaster's approach, in that it can not be reduced to the creation of new productive inputs, while holding preferences unchanged. (Bianchi, 2002). The suggested relation between innovation and preference formation is not devoid of predictive implications that will turn out relevant to the present discussion. The theme will be however taken up only in the final sections of this paper.

Until then, it will suffice for our purposes identifying the first case in our list with the creation of a new consumption good which is *not a close substitute* of any other existing good. Whether creating a 'new need', or meeting a previously unmet 'objective need', the new good is not subject to the same demand constraints that would fall upon a perfect substitute of a mature good

Variety, Consumption and Growth[*]

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Abstract

This paper attempts at fixing some guide-posts on the relation between variety, consumption and growth. A mechanism is first described, through which preference for variety expressed by intertemporally-optimizing consumers perfectly predicting the endogenously growing future consumption opportunities can cause faster steady-state growth. The mechanism amounts to a substitution of future for present consumption causing a higher steady-state savings ratio and is most naturally, but not exclusively, embedded in the intertemporal-equilibrium approach to growth modelling.

Dissatisfaction with the approach to preference for variety and innovation within the mechanism above is then motivated. The approach is oblivious of endogenous preference formation and the relation between innovation, consumption knowledge and consumption activities. Some implications concerning economic growth that follow from an evolutionary approach to these issues are then drawn.

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that is consumed in plenty and has an almost ‘saturated demand’ (low relative marginal utility). We shall assume that the new good in question is produced by a *new industry*.

The second possibility that we consider is the creation of a new consumption good which is not only radically new and meets the requirements of the first case above, but also has crucial complementarity effects. In addition to meeting a new or previously unsatisfied need, new goods open up a host of changes in the sphere of consumption, because they elicit qualitative changes in the output of other consumption industries or modify the preferences for existing goods through complementarity or external effects. Add to a car sophisticated digital instruments for audio and video communication and the experience of a driver at the wheel will not be the same as before. In this sense, the information-communication technology not only creates the home computer, but also modifies the definition of what is a car, a photographic camera, etc. and affects the utility of car-driving, picture taking, and the like. The construction-industry product innovation of supplying on a large scale non-luxury sub-urban dwellings with private garden not only brought this type of housing in the reach of the middle class, but also greatly increased the utility from having a car. Obviously enough, there are also examples of negative complementarities or externalities that may come to mind. The pleasure from shopping at the nearby grocery or from having half pint lager at the favourite pub may largely depend on the relations of acquaintance, friendship, solidarity with the clients usually met in that place; these relations, or the very possibility to meet the ‘usual clients’, may be destroyed by the diffusion of new ‘life styles’ (Earl, 1986) associated with the emergence of new goods, whether consumption goods or productive inputs. Thus, our second case is concerned with product innovations that are not perfect substitutes of any existing consumption good and, in addition, exert complementarity or external effects that increase or decrease the contribution to the personal well-being that may come from consuming traditional goods. We shall assume that also this type of innovation gives rise to a new industry.

There are of course innovations that produce close substitutes of existing consumption goods. We shall not be concerned with these innovations in the sequel, in that they are less interesting from the view-point of the long-term relation between variety, consumption and growth.

The consumption innovations considered in this paper make a non controversial case for a definite preference for variety: suppose all consumption goods bear the same price and a given composite consumption flow $C = \int_{j=0}^{n} c_j dj$ could be distributed across a larger number $n' > n$ of goods: $C = \int_{j=0}^{n'} c'_j dj$. To the extent that marginal utility is decreasing, and goods are not close substitutes of each-other, we expect that the consumer is better off after the consumption pattern has changed. The net benefit from the change in question would be even greater if consuming a larger number of goods affects in a positive direction the contribution to well being coming from consuming every single good. On the contrary, if the influence in question is negative, the net benefit from the change in consumption pattern may be partly or completely dissipated. Moreover, since consumption goods are not close substitutes and provided that the negative complementarity effects do not prevail, the availability of a larger number of goods makes a given increase in the total consumption flow *C* more desirable than it would have been the case otherwise. In this vein, the growth process is marked and sustained by the higher dynamism of the demand for the new goods, and there is a relative saturation of the demand for the old products (Kuznets, 1953; Pasinetti, 1981).

2. Technology of physical production

In the economy at time t there are n_t differentiated goods and one capital good. A differentiated good can be either consumed, or it can be used as intermediate input in capital good production. c_{jt} is the quantity of the differentiated good j consumed at t, x_{jt} is the quantity of the same good used as intermediate input at t.

Capital-good output at t is produced by perfectly competitive firms according to the constant returns to scale production function:

$$\dot{K}_t = n_t^{\frac{\alpha-1}{\alpha}} \left[\int_{j=0}^{n_t} x_{j,t}^{\alpha} d_j \right]^{\frac{1}{\alpha}} \tag{1}$$

To emphasize the response of the production system to changes in demand, it is assumed that all the inputs to production are themselves producible. There are not ‘fixed factors’ in the economy.

It is also worth stressing that to maximize the capital output-flow $\dot{K}_t$ obtained from the given total intermediate-input flow $X_t = \int_{j=0}^{n_t} x_{j,t} dj$ of n_t varieties, it is required that $x_{j,t} = x_t$, $j \in [0, n_t]$; if this is the case, $\dot{K}_t = n_t x_t = X_t$.

The functional form of (1) and competition imply that the price p_j of the intermediate input j and the price p_K of one unit of the capital good are as follows:

$$p_{j,t} = n_t^{\alpha-1} \dot{K}_t^{1-\alpha} p_{K,t} x_{j,t}^{\alpha-1} \tag{2}$$

$$p_{K,t} = n_t^{(1-\alpha)/\alpha} \left[\int_{j=0}^{n_t} p_{j,t}^{1-\varepsilon} d_j \right]^{\frac{1}{1-\varepsilon}} \tag{3}$$

where $1-\varepsilon = -\alpha/(1-\alpha)$. It is worth observing how (2) implies that the demand for the intermediate input j by competitive firms has elasticity $-\varepsilon = -1/(1-\alpha)$ with respect to p_j. A lower

α entails a less elastic demand-curve of the differentiated good j, *qua intermediate input*. Since the demand-curve of the differentiated good j, *qua consumption good*, will turn out to have price elasticity -1, a lower α is unambiguously related to a higher market power of the local monopolist producing good j.

Capital is the single physical input to differentiated-good production. Capital embodying a larger variety of ideas is not more productive. One unit of capital, if assisted by the appropriate blue-print of ideas, and no matter what is the number of intermediate-good varieties embodied therein, produces A units of differentiated goods, whatever their kind.

$$AK_{Y,t} = \int_{j=0}^{n_t} x_{j,t} dj + \int_{j=0}^{n_t} c_{j,t} dj$$

where K_Y is capital invested in physical output production.

In this sense, variety does not affect productivity and the assumption is motivated by the goal of considering the growth-effects of variety exerted through consumption demand, rather than technology and productivity.

Final output at t is $Y_t = \dot{K}_t p_{K,t} + \int_{j=0}^{n_t} c_{j,t} p_{j,t} dj$.

For the sake of later reference we observe that in a symmetric equilibrium where $p_{j,t} = p_t$, $x_{j,t} = x_t$ and $c_{j,t} = c_t$, $j \in [0, n_t]$, from (1) and (3) we obtain:

$$\dot{K}_t = n_t x_t \tag{4}$$

$$p_{K,t} = p_t \tag{5}$$

$$Y_t = p_t(\dot{K}_t + n_t c_t) \tag{6}$$

$$AK_{Y,t} = n_t(c_t + x_t) \tag{7}$$

(6) reveals that steady state investment and total-consumption expenditures grow at the rate:

$$g = \frac{\dot{n}}{n} + \frac{\dot{p}}{p} + \frac{\dot{c}}{c} \tag{8}$$

3. Preference for variety and the representative family inter-temporal plan

The representative family maximizes lifetime utility[1]

$$\max \int_{t=0}^{\infty} u_t e^{-\rho t} dt \tag{9}$$

[1] Here and elsewhere in the paper, e is understood to be the base of natural logarithms.

subject to the flow budget constraint that asset accumulation $\dot{a}$ is constrained by current income less consumption expenditure: $\dot{a}_t = r_t a_t - \int_{j=0}^{n_t} c_{j,t} p_{j,t} dj$. In this expression asset price is implicitly normalized to 1 throughout and p_j can be interpreted as the price of good j relative to asset price.

The results of the paper crucially depend on the functional form for instantaneous utility. This intends to capture the basic idea that consumers have a definite preference for variety, such that they increase their satisfaction by differentiating a given total consumption expenditure $E = \int_{j=0}^{n} c_j p_j dj$ across the highest possible number of goods consistent with the attained variety n_t and/or with the lower bound $b \geq e$ to the divisibility of goods. In particular, it is assumed:

$$u_t = n_t^{-\theta} \int_{j=0}^{n_t} \log c_{j,t} dj \; ; \tag{10}$$

$$c_j \geq b; n_0 = 1; 0 \leq \theta < 1.$$

where $(1 - \theta)$ measures the intensity of preference for variety. As stressed in the introduction, the above preference representation implicitly refers to an economy where consumption varieties are not close substitutes and the negative complementarity and externality effects of innovations do not fully dissipate the benefits from a higher differentiation of consumption. Expression (10) fully abstracts from the features, realistic as they may be, that make the contribution to instantaneous utility coming from consumption of a differentiated good, depend on the time interval elapsed since the good in question was first introduced[2]. Such features are inessential to the argument made in this paper.

Let μ_t and λ_t the discounted and undiscounted shadow price of the state variable a_t in the present-value and current-value Hamiltonian (respectively) associated to (9): $\lambda_t = e^{\rho t} \mu_t$. Necessary conditions for utility maximization are:

$$n_t^{-\theta} \lambda_t^{-1} = c_{j,t} p_{j,t} \tag{11}$$

$$\dot{\mu}_t = -\mu_t r_t \tag{12}$$

$$\lim_{t \to \infty} \mu_t a_t = 0 \tag{13}$$

(11) implies that consumption expenditure is uniform across varieties and total consumption expenditure at t is:

$$E_t = \frac{1}{\lambda_t} n_t^{1-\theta} \tag{14}$$

[2] These features are responsible of the logistic diffusion curves that are observed empirically and are explicitly introduced in Aoki and Yoshikawa (2002). As argued in section 8 below, a more thorough and satisfactory analysis of such features can be obtained only at the cost of removing the assumption of exogenous preferences, to consider the relation between novelty, preference for variety and the accumulation of consumption knowledge.

(10) and (11) yield the consumption growth equation:

$$\frac{\dot{c}_{j,t}}{c_{j,t}} = r_t - \rho - \theta \frac{\dot{n}_t}{n_t} - \frac{\dot{p}_{j,t}}{p_{j,t}} \tag{15}$$

In symmetric equilibrium with $p_{j,t} = 1, j \geq 0, t \geq 0$, (15) boils down to

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - \theta \frac{\dot{n}_t}{n_t} \tag{16}$$

4. Intertemporal equilibrium with exogenous innovations

In this section we consider some qualitative results, recently stressed in Aoki and Yoshikawa (2002), which refer to the model economy where innovations are costless, exogenous and markets are perfectly competitive. For the sake of simplicity we fully abstract from adjustment costs and their influence on capital utilization[3].

In this economy varieties grow at the exogenous rate g_n and the interest rate, as well as asset depreciation (see (12) above), is fixed by technology:

$$r_t = \frac{\dot{\mu}_t}{\mu_t} = r = A \tag{17}$$

Since in equilibrium $p_{K,t}\dot{K}_t = \dot{a}_t$, and (5) implies that in the symmetric equilibrium where $p_t = 1, 0 \leq t$, we have also $p_{K,t} = 1, 0 \leq t$ grows at the rate g_n, the transversality condition (12), together with (7), (16) and (17), imply the steady-state restriction:

$$\frac{\dot{a}}{a} = \frac{\dot{c}}{c} + g_n = A - \rho + (1-\theta) g_n < A \tag{18}$$

that is,

$$(1-\theta) g_n < \rho \tag{19}$$

In this economy all capital is invested in physical production ($K_Y = K$). Thus, for $s \equiv c/x$, (4) and (7) reveal that

$$\frac{\dot{K}_t}{K_t} = \frac{1}{A(s_t + 1)}$$

where $1/A$ and $1/(s + 1)$ can be interpreted as 'capital-output ratio' and 'savings propensity', respectively.

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[3] The influence of adjustment cost is instead prominent in Aoki and Yoshikawa (2002).

A discrete, once and for all, increase $g_n^* - g_n$ of innovation growth at time *t*, if consistent with (19), has the effect that consumers want to increase their future consumption at a faster rate. They substitute future for present consumption. This requires a higher flow of saving and investment which in a symmetric equilibrium is instantaneously achieved through a discrete rise of μ_t and a corresponding discrete fall of c_t. The economy instantaneously attains the higher steady-state (symmetric) equilibrium growth-rate $A - \rho + (1-\theta)g_n^*$.

Likewise, a once and for all increase in the preference for variety (1 − θ) does not interfere with the technologically determined interest rate *A* and, if consistent with (19), instantaneously brings the economy to a higher steady-state growth rate.

5. Intertemporal equilibrium with endogenous innovations

In this and the following sections it is assumed that new goods result from a purposeful and costly innovation effort. For the sake of simplicity, technology of the R&D sector is described by the deterministic equation

$$\dot{n}_t = \delta K_{n,t} = \delta z_{n,t} K_t \tag{20}$$

where $K_{n,t}$ is the capital stock invested in R&D and $z_n \equiv K_n / K$. Since 0 ≤ z_n ≤ 1, equation (20) implies the steady-state restriction[4]:

$$g_n = g_K \tag{21}$$

Using (21), from (4) and (7) we derive the further steady-state restriction:

$$g_x = g_c = 0 \tag{22}$$

(21), (22) and (16) yield the symmetric equilibrium, steady-state growth rate:

$$g_n = g_K = \frac{r - \rho}{\theta} \tag{23}$$

We are left with the task of studying the endogenous determination of the interest rate in this economy and its relation with the savings propensity and the allocation of capital between physical-output production and ideas production. We shall consider both steady-state and transitional equilibrium paths.

The right of producing the differentiated good *j* comes from the acquisition of the corresponding infinite-life patent, which has market value $V_{j,t}$ at time *t*. Patent acquisition

[4] Variables without the time subscript will henceforth indicate steady-state magnitudes.

represents a fixed cost for the producer of the differentiated good j, which is the local monopolist j. His flow profit $\pi_{j,t}$ is determined by current revenue $p_{j,t}(x_{j,t}+c_{j,t})$ minus flow-cost $[(x_{j,t}+c_{j,t})/A]p_{K,t}r_t$. At any date in symmetric equilibrium such that

$$p_{j,t}=p_t=1, 0\le j\le n_t, 0\le t \tag{24}$$

we have:

$$\pi_{j,t}=x_t(1+s_t)\left(1-\frac{r_t}{A}\right) \tag{25}$$

(11) implies that the price elasticity of consumption demand for the differentiated good j is –1. Using this property, in symmetric equilibrium[5] the first order condition for monopoly-profit maximization yields:

$$r_t=\frac{\alpha A}{(1-\alpha)s_t+1}$$

Since $0<\alpha<1$, and $s_t\ge 0$, $r_t<A$. Conversely, in symmetric equilibrium:

$$s_t=\frac{\alpha A-r_t}{r_t(1-\alpha)}\equiv s(r_t) \tag{26}$$

It is worth recalling that in equilibrium the fraction of income which is not consumed can be written $1/(s_t+1)$; thus the equilibrium propensity to save is fully determined by the rate of interest and we are informed by (26) that there is a positive relation between the two variables. In other words, the local monopolists' maximizing behaviour fixes the relation $1/(s(r_t)+1)$ between the interest rate at t and the equilibrium composition of output between investment and consumption at the same date. Preferences can be interpreted as affecting the equilibrium composition of output through their effect on the interest rate.

Innovation value at t is $V_{j,t}=\int_t^\infty \pi_{j,\tau}\exp(\int_t^\tau r_u du)d\tau$. In a steady-state symmetric equilibrium, $x_{j,t}=x, 0\le j, 0\le t$; steady-state innovation value can be written:

$$V_{j,t}=V=x(1+s)\left(\frac{A-r}{Ar}\right) \tag{27}$$

Capital is instantaneously transferable across sectors. Free entry in R&D implies that at any date t the rate of return on capital invested in R&D is equal to the rate of interest:

$$\frac{\dot{n}_t V_t}{K_{n,t}p_{K,t}}=r_t=\delta V_t \tag{28}$$

This yields the steady-state, symmetric equilibrium restriction $\delta V=r$, or, using (27):

[5] Symmetric equilibrium is henceforth understood to be an equilibrium such that (24) holds.

$$\delta x(1+s)\left(\frac{A-r}{Ar}\right)=r \tag{29}$$

Recalling that $K_t(1-z_{n,t})=K_{Y,t}$, from (4), (7), (20), (23) and (29) we obtain the steady-state conditions:

$$1-z_n=\frac{(r-\rho)\alpha(A-r)}{A\theta r(1-\alpha)} \tag{30}$$

$$1=\theta\frac{z_n}{(1-z_n)}\frac{r}{(A-r)}+\frac{\rho}{r}\equiv F(r) \tag{31}$$

$$\frac{K_t}{n_t}=\frac{r-\rho}{\theta\delta z_n} \tag{32}$$

Proposition 1: A economically acceptable solution r to (31) is such that A > r > ρ; $1>z_n(r)>0$. *Proof: see appendix.*

Proposition 2: Let r_1 and r_2 the real values of the interest rate that satisfy $1-z_n=1$ *in equation (30). The following inequality holds: $0 < \rho < r_1 < r_2 < A$. There exist r^* and r^{**}, $\rho < r^* < r_1$ and $r_2 < r^{**} < A$, that satisfy (31). Moreover,* $z_n(r)$ *as defined by (30) is a decreasing function of r in the interval $\rho < r < r_1$ and an increasing function of r in the interval $r_2 < r < A$, provided that r_1 is sufficiently close to ρ and r_2 is sufficiently close to A*[6]*. Proof: see appendix.*

Proposition 2 implies, among other things, that the same point in parameter space, in particular, the same state in the preference for variety and technology of R&D may be consistent with a 'low' or a 'high' value of the interest and growth rates. Moreover, the proposition suggests that these high or low rates may not map in a straightforward way to the share of resources invested in R&D.

Proposition 3 (Transitional dynamics): Let r be a steady-state interest rate identified in proposition 2 and consider the corresponding steady state path of the economy. At the initial date t = 0 the stocks K_0 *and* n_0 *are pre-determined and fix a transitional-equilibrium path converging to steady-state, with the following properties.*

$$r_t=r, 0\le t; V_t=r/\delta, 0\le t$$

$$z_{n,t}=1-\frac{r^2}{(A-r)\delta}\frac{n_t}{K_t}$$

[6] A sufficient condition for this is that $A\theta(1-\alpha)/\alpha$ is sufficiently small.

$$\frac{\dot{n}_t}{n_t} = \delta \frac{K_t}{n_t} - \frac{r^2}{A-r}$$

$$\frac{\dot{K}_t}{K_t} = \frac{A}{(1+s)} \; \frac{r^2}{(A-r)\delta} \frac{n_t}{K_t}$$

$$\frac{\dot{c}_t}{c_t} = r - \rho - \theta \left(\delta \frac{K_t}{n_t} - \frac{r^2}{A-r} \right)$$

Proof: see appendix.

Proposition 3 implies that the ratio between the stocks of physical capital and ideas, from any arbitrarily given initial condition converges monotonically to its steady-state value determined by (32). On the assumption that the initial value of this ratio is higher than at steady state, then the growth rate of physical capital $g_{K,t}$ converges to $g = (r-\rho)/\theta$ from below and the growth rate of ideas $g_{n,t}$ converges to $g = (r-\rho)/\theta$ from above. During the transition, the share of resources invested in R&D is larger than at steady state. In other words, this share converges to its steady-state value from above.

Although proposition 2 points to the possibility of multiple steady-state equilibria, the remark that $\lim_{\rho \to 0} r_1 = 0, \lim_{\rho \to 0} r_2 = A\left[1-\theta(1-\alpha)/\alpha\right]$ shows that by assuming ρ sufficiently close to zero we can plausibly restrict the equilibrium interest rate to the interval $\left(r_2, A\right)$. With this restriction in mind, we state the following propositions.

Proposition 4 (Comparative-statics effects of a change in the preference for variety): Fix a given point $\mathbf{q} = (\alpha, \delta, \theta, \rho)$ *in parameter space, such that* ρ *is sufficiently close to zero, and consider the steady-state effects of the parameter change* $\mathbf{q}' - \mathbf{q} = (0, 0, \theta' - \theta, 0)$, $\theta' - \theta < 0$, *where both* $\mathbf{q}'$ *and* $\mathbf{q}$ *meet the restriction imposed by transversality. Consider the interest rates* $r(\mathbf{q})$ *and* $r(\mathbf{q}')$ *identified by proposition 3. We obtain:*

$$r(\mathbf{q}') > r(\mathbf{q})$$

$$g(\mathbf{q}') = \frac{r(\mathbf{q}') - \rho}{\theta'} > \frac{r(\mathbf{q}) - \rho}{\theta} = g(\mathbf{q})$$

The effects on the fraction of capital invested in R&D are more ambiguous, because, ceteris paribus, the parametric fall of θ *tends to lower* z_n*, but the consequent rise of the interest rate tends to raise* z_n*. Proof: see appendix.*

Proposition 5 (Comparative-statics effects of a change in the productivity of R&D): Fix a given point $\mathbf{q} = (\alpha, \delta, \theta, \rho)$ *in parameter space, such that* ρ *is sufficiently close to zero, and consider the* *steady-state effects of the parameter change* $\mathbf{q}° - \mathbf{q} = (0, \delta° - \delta, 0, 0)$, $\delta° - \delta > 0$, *where both* $\mathbf{q}°$ *and* $\mathbf{q}$ *meet the restriction imposed by the transversality condition. Consider the steady-state interest rates r(*$\mathbf{q}$*) and r(*$\mathbf{q}°$*) identified by proposition 3. We obtain:*

$$r(\mathbf{q}°) = r(\mathbf{q}); z_n(\mathbf{q}°) = z_n(\mathbf{q})$$

$$\frac{K_t(\mathbf{q}°)}{n_t(\mathbf{q}°)} = \frac{\delta}{\delta°}\frac{K_t(\mathbf{q})}{n_t(\mathbf{q})}$$

Proof: Direct inspection of (30), (31) and (32) yields the stated results .

Proposition 6 (Comparative-statics effects of a change in market power): Fix a given point $\mathbf{q} = (\alpha, \delta, \theta, \rho)$ *in parameter space, such that* ρ *is sufficiently close to zero, and consider the steady-state effects of the parameter change* $\mathbf{q}^{\S} - \mathbf{q} = (\alpha^{\S} - \alpha, 0, 0, 0), \alpha^{\S} - \alpha < 0$, *where both* $\mathbf{q}^{\S}$ *and* $\mathbf{q}$ *meet the restriction imposed by the transversality condition. Consider the steady-state interest rates r(*$\mathbf{q}$*) and* $r(\mathbf{q}^{\S})$ *identified by proposition 3. We obtain:*

$$r(\mathbf{q}^{\S}) < r(\mathbf{q}); z_n(\mathbf{q}^{\S}) > z_n(\mathbf{q})$$

Proof: see appendix.

Proposition 6 shows how (at a low rate of time preference) a higher degree of market power is conducive to a higher share of resources invested in R&D and to a lower rate of steady growth.

6. Remarks on more general technology and preference assumptions

A special feature of the model outlined in sections 3, 4 and 5 is that leisure does not enter the utility function and capital is the only input to production in both the R&D and physical-output sectors. The special feature moulds the stated effects of preference for variety in various ways, but two in particular are worth emphasizing here.

The first implication is that the intra-temporal substitution effects of preference for variety are confined to set of consumption goods existing at the same date and are for this reason separable from the inter-temporal substitution effects. If instead well being depends also on leisure, preference for variety would generally affect the rate at which agents are prepared to substitute at any given date the current consumption of goods for the current consumption of leisure. With labour entering the physical-output and R&D production technologies, this intra-temporal substitution effect would also have inter-temporal repercussions. The above separability between intra-temporal and inter-temporal substitution effects would not be any longer at hand.

The second implication is concerned with the conditions enabling the existence of a steady state. With leisure entering the utility function, the implied form of non separability that was just considered has the consequence that the existence of a steady state may require further ad-hoc restrictions on technology and on the way in which the consumption variety available at a given date affects the marginal utility of physical consumption relative to the marginal utility of leisure (see appendix A.2 for an example).

7. Fix-price inter-temporal equilibrium with endogenous innovations

In this section we introduce crucial modifications to the model outlined in section 5, while retaining the same assumptions on technology and market structure.

The crucial modifications are that the interest rate is exogenously fixed (we may think of the monetary authority controlling the level of *r*), the assumption of perfect foresight is dispensed with. As a result, the role of inter-temporal preferences will require further examination.

At any date *t* consumers' choices are consistent with maximization of u_t (specified by (10) and reflecting a preference for variety), subject to the consumption-goods prices $p_{j,t}, 0 \leq j \leq n_t$ and to a consumption budget E_t resulting from the time-*t*-updating of their inter-temporal choices. Our presentation will first leave the restrictions resulting from these inter-temporal choices in the background, with the aim of stressing that, at the exogenous interest rate *r*, restrictions from technology, profit maximization and arbitrage are sufficient to determine the conditions for steady-state growth. Inter-temporal choices will be brought back again in the final part of this section and more thoroughly discussed in the next.

As before, the specification (10) implies that consumption of good *j* has price-elasticity − 1 and in symmetric equilibrium at *t* the consumption budget $E_t = \int_{j=0}^{n} c_{j,t} p_{j,t} dj$ is uniformly distributed across the n_t goods. Moreover, in the present simplified framework of no physical capital depreciation, no adjustment costs and constant prices, the user-cost of capital is the interest rate. In turn, this brings with it an unmodified symmetric-equilibrium relation (26) between *s* and *r*, as a result of the local-monopolist's profit maximization. If in the model of section 5 *s* and *r* were simultaneously determined in equilibrium, now the exogenously fixed interest rate and the macroeconomic symmetric equilibrium relations fix the 'propensity to save' $1/(s+1)$ as well, leaving the consumer with no degree of freedom in this respect. In the present framework, consumer's preferences can impinge on the equilibrium consumption/output ratio only at the cost of making the interest rate endogenous.

Aggregate capital accumulation is determined through an accelerator-type equation that can be easily 'micro-founded'.

$$\dot{K}_t = \frac{1}{A} g^e_X n_t (c_t + x_t) + \frac{1}{\delta} g^e_n \dot{n}_t \qquad (33)$$

where g^e_y is the uniformly expected growth rate of the variable y and $X \equiv \int_{j=0}^{n} (c_j + x_j) dj$ is the aggregate demand for differentiated goods.

Restricting (7) to steady state, differentiating with respect to time and using (4) we obtain:

$$g_n (1+s) = A(1 - z_n) \qquad (34)$$

Substituting from (4), (34) and (29) into (33) and using the steady-state, warranted-growth condition $g^e_X = g^e_n = g_n = g_K$ we obtain the following expression of the warranted growth rate g:

$$g^2 (1+s) \frac{A-r}{r^2} + g(1+s) - A = 0 \qquad (35)$$

$$g = \frac{-r + \sqrt{r^2 + 4Ar(1-\alpha)/\alpha}}{2(1+s(r))} \equiv g(r) \qquad (36)$$

where the function *s(r)* is defined by (26).

Notice that $g(r)$ is increasing in its argument. To be consistent with steady-state-equilibrium, growth expectations must be positively tuned with the exogenous interest rate, for this positively affects both the output $x(1+s)$ of each differentiated good and the investment share of this output which simultaneously preserves the ongoing equilibrium on the goods markets and the full capital-stock utilization in material and non-material production[7].

The steady- state capital share invested in R&D is

$$z_n = 1 - g \frac{1+s(r)}{A}$$

It is immediate consequence of (36) that at higher steady-growth rates of final output and of the number of varieties, a lower share of resources is invested in R&D. The result is related to the particular technology assumed for the R&D sector, which is extremely intensive in the input produced by the final output sector.

A further remarkable feature of (36) is its complete independence of the preference for variety θ, and indeed of any preference parameter whatsoever. Preferences conjure to arrive at the result (36) only in that the demand for consumption good *j* has elasticity – 1 with respect to p_j and the expenditure on each consumption good is uniform[8]. As stressed in the previous section, these

[7] We may also notice, in passing, how at the given interest rate r the relation between the growth rate g and the output/capital ratio A (in the differentiated good sector) is ambiguous, because a parametric rise of the latter increases the equilibrium value of s, as determined by the function $s(r)$ (see (26) above).

[8] It is related to preference for variety and the shadow price of capital by (11).

features, together with the local monopolists' maximizing behaviour, fixes the relation $1/(1+s(r))$ between the interest rate and the symmetric-equilibrium composition of output between investment and consumption. Whereas in the previous section the market effects of consumers' choices conjured to determine the equilibrium interest rate, now, to the extent that *r* is exogenous, preferences do not have the same scope for action. But since equilibrium must include the notion that agents are satisfied with what they are doing, we are forced to conclude that a state of equilibrium will be one in which the interest rate management is appropriately tuned with consumers' preferences. The interest rate ceases to be exogenous and exogenous preferences apparently re-emerge as the prime mover.

To illustrate this point, let me suppose, that the assumption of perfect foresight ruling in section 5 is now temporarily replaced with subjectively certain expectations. The illustrative, thought-experiment nature of the exercise is worth emphasising. In fact, the assumption of subjectively certain expectations seems particularly unsuited to the strongly Harrodian flavour of the model outlined in this section. Harrod himself was inclined to hold the view that the distant future is 'violently uncertain'[9]. Having thus stated the necessary qualifications, let me assume that consumers formulate subjectively-certain expectations on choice sets and parameters at all future dates and, on this ground, hold to the objective functional (9). If at the exogenous interest rate *r* the warranted growth rate determined by (36) and (26) does not happen to coincide with the unique steady-growth rate consistent with consumer optimising behaviour, namely $r-\rho/\theta$, then a full steady-state equilibrium does not exist. Indeed, on a steady-growth path like (36) fixed by an exogenously given interest rate, consumers correctly forecasting $g_X^e = g_n^e = g$ would not be generally satisfied with what they are doing[10]. Thus, full steady-state equilibrium entails the simultaneous fulfilment of a twofold knife-edge condition. Economic agents are required to correctly forecast the growth rate *g,* prices, and the characteristics of future commodities; the monetary authority to choose the 'appropriate' level of the interest rate. We may observe, in passing, how a higher preference for variety, though rising the warranted growth rate under such ideal conditions, may still not be able to accelerate the actual growth of the economy, because Harrod's considerations on the local instability of growth expectations in the neighbourhood of *g(r)* seem to apply.

8. Preference for variety re-defined and its diverse demand effects.

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[9] Cf. Harrod (1971), pp. 175-76.

[10] They would not be expanding their consumption at the desired rate $\dot{c}_t/c_t = r-\rho-\theta g$ unless, possibly by a fluke, $r-\rho-\theta g = 0$.

The crucial characteristic of preference for variety, as outlined in section 3, is that it is a well defined preference ordering over a time-varying choice set, which is known ex-ante. The approach, as further examined in sections 4, 5 and 7, brings to the fore the following implications for equilibrium growth:

(i) At any given interest rate r_t and growth rate of varieties $g_{n,t}$ a higher preference for variety (lower θ) causes a higher desired growth rate of consumption (16) of each differentiated good, and a corresponding higher savings flow at t, because the optimising agent prefers to postpone consumption at dates in which she will be able to benefit from the opportunity of a wider choice set. The particular model structure separates the above inter-temporal-substitution effect from other intra-temporal substitution effects that arise when leisure affects well being and labour is an input to technology (see section 6 and appendix A.2).

(ii) In economies where new goods are the outcome of R&D effort and differentiated goods are produced by local monopolists, preference for variety and profit maximization impose tight restrictions on the relation between the consumption/output ratio and the rate of interest. The relation depends on the degree of product-market competition in the differentiated good sector. In this sense, market power has a direct bearing on the equilibrium propensity to save.

(iii) In the full-fledged general equilibrium model with complete markets and small rate of impatience a greater preference for variety causes faster growth both in the level of the capital stock and in the number of goods, higher steady-state output of each differentiated good and higher investment-output ratio. In the fix-price economy driven by subjectively-certain expectations such growth enhancing effects of preference for variety would be contingent upon the correct choice of the interest rate by the monetary authority and the correct expectation formation by the agents.

As a matter of interpretation, a stronger preference for differentiation in consumption, corresponding to a lower level of the parameter θ, can be thought of as resulting from more radical qualitative differences between goods, hence from a higher and perfectly foreseen novelty content of the innovation flow. Surprise, learning and endogenous preference formation, that are so characteristic of consumption innovation, are ruled out by definition from the above representation. In this sense we are inclined to interpret the effects from *(i)* to *(iii)* as related to the *foreseen component* of variety growth. In a long-term framework innovation phenomena become part of a normal state of affairs and up to some extent their effects can be predicted. But it is a logical consequence of innovation as a carrier of true novelty that there must be a large *unforeseen component* of variety growth. In what follows we expand on some effects of the latter that are relevant to the relation between consumption and growth.

The formation of preferences for truly new goods entails learning and knowledge acquisition processes that mostly occur in the course of consumption activities (Bianchi, 1998; Loasby, 1998; Scitovsky, 1992; Swann, 1999; Witt, 2001) or of interactions with other heterogeneous consumers (Dosi et al., 1999) and in any case not before the relevant information or reinforcement signals are released. Preference for variety is often the outcome of an experience-based discovery of consumption complementarities (Bianchi, 1998, 2002). To this extent, consumers mostly become aware of their preference for variety only after the new goods are marketed. Self-perception of preference for variety entails surprise and its effects can not be adequately recounted within the strait jacket of an equilibrium framework where, paradoxically, novelty is fully anticipated.

A consumer who is truly and favourably surprised at time t_0 by the acquired consumption knowledge on the number and service characteristics of the new goods available, and who is not expecting further favourable surprises in the future, may wish doing more than simply modifying the planned composition of her consumption basket (partly substituting the new goods for the old ones). She would wish at time t_0 to increase her consumption at dates close to t_0 over and above what she had planned to do on the base of the wrong perception that such a wider and attractive consumption differentiation would be available only in a more distant future.

The argument above suggests that the demand effects of a variety-innovation flow will largely depend upon the prevailing foreseen or unforeseen nature of the flow. Unforeseen substitution effects are triggered by the diffusion within the population of agents of the knowledge about the consumption opportunities disclosed by innovations that have already taken place. The consumers newly reached by the diffusion process have both motives and knowledge for formulating a new inter-temporal consumption plan, conditional on their current information set, and on the awareness that further surprises may arrive in the future.

In a world where consumers are generally aware that they may be unaware of the future consumption opportunities and preferences are endogenously shaped through the processes concomitant to the introduction and dissemination of novelty, the inter-temporal substitution effect summarized under *(i)* above lacks the necessary knowledge requirements. We argue that the reliability of predictions concerning the detailed qualitative characteristics of the innovation flow in the distant future is bound to be low, thus inducing consumers to not give much weight to their subjective far-reaching aniticipations of consumption knowledge. The inter-temporal optimisation, rational-choice toolbox may be suited to represent the effects of routine behaviour in a steady environment (Loasby, 2001), but is little suited to consider consumers' reactions in the face of true novelty. A relevant implication is that formal conditions such as the Euler equation (16) may offer a misleading description of the inter-temporal substitution effects induced by variety, if anything because they assign too much weight to the distant future.

The emphasis on consumption knowledge and the suggested separation between the foreseen and unforeseen components of the innovation flow brings to the fore the relevance to the present discussion of the information distribution concerning the qualitative and quantitative features of innovations. The issue could not even arise in the perfect information framework of section 3, with the outcome that the full-equilibrium relation between preference for variety and growth was formally the same for flex-price and fix-price models. If in that framework we could quite innocently disregard the asymmetric position held on the demand side by consumers, innovating and non-innovating firms, now this abstraction becomes untenable. It is now crucial to posit that the additional demand for physical-capital comes from innovating firms, that is, the holders of the information concerning the nature and service characteristics of the new goods just marketed. Recalling that the strength of preference for variety is related to the novelty content of consumption innovations and to consumption complementarities, we can see that the innovating firm is normally[11] in a better position to correctly predict the demand for the variety which it is about to introduce.

To this extent, we argue that a higher rate of variety-innovation effort induces a higher investment/output ratio through the investment demand by the innovating firms, but there is no sound reason for assuming that the higher investment flow is matched by a higher flow of *planned* savings justified by consumers' willingness to postpone their consumption in the face of the correctly predicted future expansion of the choice set. In fact, the set of ideas referred to in this section is not easily reconciled with the equilibrium approach to growth-modelling, but points to an evolutionary approach where surprise, learning, heterogeneity and social interaction can play a major role. Moreover, there are difficulties inherent to the project of subsuming the processes characterized by the emergence of true novelty under the discipline of a set of mathematical tools mapping every relevant dimension of change[12]. Thus, developing an evolutionary theory of long-

[11] The qualification is needed to account for cases in which innovating firms were quite blind to the consumer-demand opportunities open to their innovations. The radio and the telephone are cases in point (see Metcalfe, 2001, p. 47 and the references there cited).

[12] It is not denied here that there are dimensions of evolutionary change that lend themselves to some formal analysis. These dimensions are not exclusively confined to the dissemination of an exogenously given novelty, under the assumption "that no further novelty will intervene in the post-revelation analysis", as argued in Witt (2003). They extend, for instance, to the study of the structural constraints faced by evolutionary change in general, as shown by the recent trans-disciplinary literature on modularity and network evolution.

term economic growth is not conducive to a single general formal model, but to a set of models that are connected by a way of reasoning about change. Here, we note a single but important point of convergence between the suggested 'view' of the economic process and a non-equilibrium interpretation of the model outlined in section 6. The convergence point is related to the already emphasized fundamental knowledge-asymmetry between innovating firms and consumers.

9. Conclusions

A higher innovation effort may well be justified by innovators' correct anticipation of consumers' preference for a certain class of varieties. In this sense, the argument above suggests that the macroeconomic effects of preference for variety are expansionary, in that they sustain investment expenditure. There are also crucial effects on the decisions concerning the size and composition of consumption in the near future, conditional on the available information set. Far reaching inter-temporal substitution effects are more ambiguous, because the predicted and unpredicted components of variety growth may act in opposite directions, and because long-term predictions of variety growth are unreliable.

Moreover, we were able to show that there are tight restrictions linking the interest rate at t with the output composition between investment and consumption at the same date. This suggests that, to be effective, the alleged expansionary effects of variety growth must be favoured by a skill-full interest-rate management by the monetary authority keeping its policy in tune with the optimistic entrepreneurial expectations. In this case, we would have that the 'equilibrium growth rate' keeps track of the actual growth rate.

The question remains if a faster expansion of variety can increase the growth rate of output over long-lasting time intervals, and independently of any further effect on input-productivity. For reasons of analytical convenience, persistent changes in the growth rate are normally studied by referring to steady-state comparative dynamics. This was also the approach used in sections 4, 5 and, if only for illustrative purposes, a small part of section 7. In the stated analytical frameworks, the growth effects of variety act through the inter-temporal substitution of future for present consumption. We explained at length in section 8 why such inter-temporal effects, at least in as far they are linked to variety growth, are uncertain and are not likely to be strong. Are we to conclude, on this ground, that in itself the expansion of consumption variety does not have robust growth effects of a persistent nature?

There are at least two lines of reasoning suggesting that such a conclusion would be unconvincing. The first and general reason is that the identification between permanent-growth effects and changes of steady-state outcomes is objectionable (Temple, 2003). A systematic upward pressure on the ex-post investment-output ratio from a non sporadic faster creation and growth of new goods and industries would affect the actual, non equilibrium growth path over those time intervals that are normally in the focus of the growth theorist. This may well occur notwithstanding that no comparably sizable change takes place in what it was defined before 'the perfectly foreseen component' of variety growth. I shall not expand on this point here[13].

The second reason is that in a more general setting the inclusion of the labour input and of leisure would bring with it new forms of intra-temporal and inter-temporal substitution. If the previous objections to the tight link between consumption variety and far-reaching inter-temporal substitution do not seem to loose their weight, now, growth effects can occur through the intra-temporal substitution between leisure, labour and consumption. In particular, consumption variety would affect life styles, the stringency of income and time constraints on consumption activities (Metcalfe, 2001) and the arbitraging[14] between the benefits from goods and leisure consumption on one hand, and between leisure and labour effort on the other. The ensuing effects on labour supply, the propensity to consume and the allocation of resources to the output and R&D sectors do not need to rest on the perfect anticipation by consumers of the novelty content of the new goods available in the far distant future.

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[13] Temple (2003) insists that so-called level effects may well be long-lasting and empirically more relevant, for reasoning about growth, than steady-state results based on knife-edge restrictions. As long as we do away with the assumption of full utilization of resources, we may extend Temple's argument, to include the possibility of long-lasting deviations from the fully anticipated component of growth paths.

[14] Matcalfe (2001, p. 55) convincingly argues that some form of arbitrage condition will not cease to describe the equilibrium pattern of consumer behaviour after the implications of bounded rationality are explicitly considered.

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Appendix A.1

Proof of Proposition 1: We recall that the function $z_n(r)$ is defined by equation (30). $r = \rho$ implies $z_n(r) = 1$, which is inconsistent with (31). $r < \rho$ is not acceptable, because it yields $z_n(r) > 1$. $r = A$ inconsistent with (31) and $r \geq A$ implies $z_n(r) \geq 1$. r solution to (31) and $z_n(r) = 0$ implies a contradiction since $z_n(r) = 0$ is consistent with (31) only if $r = \rho$, but $z_n(\rho) = 1$.

Proof of Proposition 2: r_1 *and* r_2 are the real solutions to the equation $\frac{(r-\rho)\alpha(A-r)}{A\theta r(1-\alpha)} = 0$. It is easily seen that they meet the stated restrictions. Let us compute the derivative: $\frac{d(1-z_n(r))}{dr} = \frac{\alpha}{(1-\alpha)A\theta}\frac{(A\rho - r^2)}{r^2}$. This proves the stated properties of the function $z_n(r)$. The function $F(r)$ defined by (31) is continuous in the intervals (ρ, r_1) and (r_2, A). Moreover, we have: $F(\rho) = +\infty$ and $F(r_1) < 1$. $F(r_2) < 1$ and F(A) = +∞.

Proof of Proposition 3: Using $\dot{K}_t + n_t c_t = A(1 - z_{n,t})K_t$ together with the equilibrium equality between income $(K_t + n_t V_t)r_t$ and expenditure $\dot{K}_t + n_t c_t + \dot{n}_t V_t$ and (28) we obtain:

$$(1 - z_{n,t}) = \frac{n_t}{K_t}\frac{r_t^2}{\delta(A - r_t)} = \frac{\dot{K}_t}{K_t}\frac{(1 + s_t)}{A} \tag{37}$$

Using the above results and (25) we obtain that at any date *t* in symmetric equilibrium:

$$\pi_{j,t} = \frac{r_t^2}{\delta} \tag{38}$$

Substituting from (38) into the asset equation $\dot{V}_t = -\pi_{j,t} + r_t V_t$ it yields that at any date *t* in symmetric equilibrium:

$$\dot{r}_t = 0; \dot{V}_t = 0; \dot{s}_t = 0 \tag{39}$$

n_t and K_t are predetermined at any date *t*. Thus, using (37) and (39) we derive the transition paths for $z_{n,t}$, n_t and K_t:

$$z_{n,t} = 1 - \frac{r}{A - r}\frac{1}{\delta}\frac{n_t}{K_t} \tag{40}$$

$$\frac{\dot{n}_t}{n_t} = \delta\frac{K_t}{n_t} - \frac{r}{A - r} \tag{41}$$

$$\frac{\dot{K}_t}{K_t} = \frac{A}{(1 + s)}\frac{r^2}{(A - r)}\frac{1}{\delta}\frac{n_t}{K_t} \tag{42}$$

Proof of Proposition 4: We compute: $\lim_{\rho \to 0} r_1 = 0; \lim_{\rho \to 0} r_2 = A\left[1 - \theta\frac{(1 - \alpha)}{\alpha}\right]$. On the assumption that *ρ* is sufficiently close to zero and the steady-state interest rate *r* = *r***, that is $r_2 < r < A$, the steady-state share $z_n(r)$ is increasing in *r*. Inspection of (30) shows that a parametric fall of θ causes a fall of $z_n(r)$ at given *r*. This, together with the continuity of $F(r)$, proves the proposition.

Proof of Proposition 6: (30) implies that a ceteris-paribus parametric fall of α causes a discrete rise of $z_n(r)$ at given *r*, that is, $z_n(\mathbf{q}^{\S}, r) > z_n(\mathbf{q}, r)$ at given *r*. Since $F(r)$ is increasing in its argument in the interval (r_2, *A*), under the stated assumptions, it follows that $r(\mathbf{q}^{\S}) < r(\mathbf{q})$. Moreover, at *ρ* sufficiently close to zero, it must be the case that $z_n(\mathbf{q}^{\S}) \equiv z_n(\mathbf{q}^{\S}, r(\mathbf{q}^{\S})) > z_n(\mathbf{q}, r(\mathbf{q})) \equiv z_n(\mathbf{q})$.

Appendix A.2: To be completed.