Education and Poverty in a Solow Growth Model[*]

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Abstract

In this paper, we study the effect of education on economic growth. In particular, we show that education can generate nonlinearities in the process of human capital accumulation, which affects the economic growth path. In our model of human capital accumulation, a non constant human capital obsolescence rate can cause non constant returns to scale of education in the production of human capital. We identify the conditions for this to cause multiple equilibria in a Solow growth model. Furthermore, we calibrate our model to give reasonable values of parameters, to have multiple steady states. In the second part of this work, we will conduct some econometric analyses to prove that the returns to scale in producing human capital are non constant.

Keywords: Economic growth, human capital accumulation, multiple equilibria.

JEL: E13, I2, O4.

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1 Introduction

Since the 1960s, a large body of research has been produced on the topic of education and economic growth, but only in the last ten years the flow of statistical analyses on this subject has increased considerably. This is due to the relatively recent availability of aggregate data on country-level education.

In macroeconomic literature, education is considered one of the most important inputs to produce human capital, which can be defined as: "the stock of accumulated skills and experience that make workers more productive," [Stiglitz and Boadway (1994)]. Traditionally, human capital does not play an explicit role in the neoclassical growth theory, while it is central in the endogenous growth theory.[1] Nevertheless, the neoclassical model has been recently extended to the inclusion of human capital by Mankiw, Romer and Weil (hereafter MRW) (1992). By using a cross-country analysis, they show that data are fairly consistent with a Solow model augmented to take into account human capital as a factor of production. They obtain a rather satisfactory estimate of the aggregate production function. In this framework, education does not produce externalities at the aggregate level. That is, education appears as a private input, which is remunerated according to its marginal product. Barro and Sala-i-Martin (1995), through an extensive test on cross-country data, show that the neoclassical model could explain several empirical facts.

Following Aghion and Howitt (1998), we can divide the macroeconomic literature on the relationship between education and growth into two branches. The first branch considers the process of *human capital accumulation* over time to explain long-run growth. This literature is based essentially on the seminal works of Uzawa (1965) and Lucas (1988). In particular, Lucas (1988) provides a growth model in which the process of human capital accumulation is the fundamental factor to explain the long-run growth. Lucas shows that, in a model with human and physical capital, differences across countries in the mechanism of human capital accumulation determine large and persistent differences in their growth rates. The second branch considers the *stock of human capital* as a determinant of growth. This approach is based on contributions such as Nelson and Phelps (1966) and Romer (1990). Nelson and Phelps (1966) argue that education plays a fundamental role in economic growth facilitating the process of technological diffusion. They show that high levels of human capital are necessary to sustain high rates of technological progress. Romer (1990) builds an endogenous growth model, where human capital is an input either in a traditional production sector or

[1] The first examples of neoclassical growth models in which economic growth is exogenous can be found in Solow (1956) and Swan (1956). While, Lucas (1988) and Romer (1990) provide two important endogenous growth models in which human capital plays a central role in the process of economic growth.

in a R&D sector. There, technological progress is a nonrival, partially excludable good, which leads to monopolistic competition in the market of final goods. The main implication of Romer's model is that the stock of human capital devoted to the research sector determines the rate of economic growth. Unfortunately, the empirical literature cannot help to completely understand which approach is more appropriate: some studies suggest that both the variation and the stock of human capital can explain GDP growth, while other works show that only the stock of human capital has a significant effect on economic growth.

A traditional assumption in the theory of economic growth is that human capital is produced under constant returns to scale, using education as a single input.[2] Yet, there is no compelling evidence to support this assumption. On the contrary, an increasing number of analyses shows that the production of human capital exhibits increasing returns to scale for low levels of education and decreasing returns to scale for high levels of education. For instance, Krueger and Lindahl (2001) find evidence in favor of an inverted-U shaped relationship between the stock of human capital and the GDP growth rate. By comparing different regression models, they find that the best fitting of the data is provided by a regression model that considers a quadratic form for education. The inverted-U pattern suggests that there are increasing returns to education only for countries with a low level of education (below 7.5 average years of schooling), while for countries with high levels of education (above 7.5 average years of schooling) returns to education are decreasing. By analyzing the effect of human capital in an open economy, Isaksson (2002) confirms the Krueger and Lindahl's result concerning the existence of a nonlinear relationship between education and economic growth. By introducing a measure for trade openness, Isaksson finds also important interaction effects between education and trade openness when education enters in a nonlinear fashion. Trostel (2004) obtains the same results, by using an international micro-dataset to estimate an aggregate Mincerian equation [from Mincer (1974)]. There, evidence indicates that the production function of human capital displays increasing returns at low levels of education and decreasing returns at high levels of education. Moreover, these nonlinearities occur primarily within countries. This means that these nonlinearities could be a direct consequence of the process of human capital accumulation.

In this paper, we introduce a possible mechanism to explain the nonlinear relationship between education and human capital and then education and economic growth. In particular, we consider the effect on GDP growth of a non constant depreciation rate of human capital. Specifically, we assume that the depreciation rate of human capital is *positively related* to the level of education attained by an

[2] In other words, "units" of human capital (H) are assumed to be in a one-to-one relation with "units" of education (e), that is $H = e$.

individual.[3] Here, as in standard literature, education is a fundamental input to produce human capital. Nevertheless, with respect to the standard theory, we do not assume a priori the existence of constant returns to scale in producing human capital through education. We provide a theoretical framework in which education can generate a nonlinear process of human capital accumulation. We will then use this result to modify the Solow model. Subsequently, we will show that our model can explain the existence of multiple equilibria in the output growth path. Through a numerical example, we will show that, for reasonable values of the parameters, our model may generate multiple steady-states.

In the second part of the paper, we use a cross-section of 78 countries to conduct some econometric analyses. The aim of the empirical analysis is to show that the returns to scale in producing human capital are not constant. To show that the level of schooling has a nonlinear effect on GDP growth rate, we will compare a traditional regression method with a semiparametric technique also used in Liu and Stengos (1999). As we will see, evidence is consistent with our theoretical model.

The paper is organized as follows: Section 2 contains our modified Solow growth model; Section 3 provides a numerical example to prove the aforementioned main result and its consistency with actual values of the parameters; Section 4 reports the econometric analysis; Section 5 illustrates the main conclusions of this work.

2 The Model

In this section, we propose a growth model in which the process of human capital accumulation can explain the existence of nonlinearities in the economic growth path. To do this, we divide the model into two parts. In the first part of the model, we show how, through the acquisition of formal education, individuals can accumulate human capital in a nonlinear way. In the second part of the model, we put this result into a traditional Solow growth model in order to study the consequences of these nonlinearities on the aggregate level of output.

2.1 The Process of Human Capital Accumulation

Consider a closed economy in which markets are competitive and economic activity is performed over continuous time. Time is indexed by t, and individuals live for an infinite time horizon. Let L_t be the mass of population at time t, and assume that *agent* $i \in [0, L_t]$ allocates his or her lifetime among work, education, and leisure. Let u be the constant fraction of time that each individual devotes to work, while

[3] This assumption is supported by several studies. For a rather detailed review of these studies see de Grip (2004).

$e_{i,t}$ denotes the amount of time that agent i has already invested in education at time t.[4] That is, we can write:

$$e_{i,t} = \alpha_i t \tag{1}$$

where $\alpha_i \in [0,1]$ indicates the fraction of time that individual i invests in education and can be considered as a measure of the individual propensity to study.[5] In this section, we do not make any particular assumption on the determinants of α_i. While, in the next section, following studies such as Bils and Klenow (2000) and Glewwe and Jacoby (2004), we will assume that the average level of education observed in a country depends on the country's stock of physical capital per capita.[6]

Following the standard literature on human capital and economic growth, we consider education as the only input to produce human capital. Nevertheless, with respect to the standard theory, we do not assume a priori the existence of constant returns to scale in producing human capital. Therefore, using a general human capital production function for individual i, we have:

$$h_{i,t} = \phi(e_{i,t}), \quad \phi' = \frac{dh_{i,t}}{de_{i,t}} > 0 \tag{2}$$

where $h_{i,t}$ is the stock of human capital owned by agent i at time t, and $\phi(.)$ is an unknown function. Equation (2) only states that the individual stock of human capital depends on the time already invested by individual in education at time t. Nevertheless, this equation does not explicitly describe the relationship between education and human capital. In order to obtain a more specific formulation for Equation (2), we consider the mechanism through which individuals accumulate human capital. Therefore, $h_{i,t}$ will be the result of this accumulation process.

Human capital variation is determined by the difference between the human capital created in a given period and the human capital destroyed in the same period. Since the individual's stock of human capital may facilitate the acquisition of further knowledge, the accumulation of human capital will depend on the stock of human capital and on the productivity of the education sector, as in Lucas (1988). At the same time, the destruction of human capital will depend on the quantity of human capital subject to obsolescence, a feature not considered by Lucas (1988).

[4] We astract from the issue of optimal choice of allocation of time among work, education and leisure.

[5] Obviously, the amount of time that individual i has already spent in liesure at time t will be $(1 - \alpha_i - u)t$.

[6] In this way, as suggested by Glewwe and Jacoby (2004), we are implicitly assuming that the value of α_i depends on the individual stock of physical capital as well as other individual factors such as preferences for education, aptitude, etc.

We assume that the depreciation rate of human capital increases as the stock of human capital increases.[7] By analyzing the effect of technological change on schooling-specific obsolescence, that is the obsolescence of skills acquired at school, various papers provide theoretical and empirical arguments in favor of this hypothesis. According to these studies, technological progress is one of the major causes of skill obsolescence. This is due to the fact that new technologies require the acquisition of new knowledge, which replaces part of the existing knowledge. Rosen (1976), Weiss and Lillard (1978), and Johnson (1980) show that technological change increases the depreciation rate of human capital, that is the knowledge embodied in individuals. Therefore, the higher the rate of technological innovation, the higher the depreciation rate of human capital will be. Two causes may justify the assumption of an increasing depreciation rate of human capital:

1. The first cause is known as "vintage effect" and is well illustrated in Neumann and Weiss (1995). There, they give evidence that high skilled workers are more affected by depreciation of human capital than low skilled workers. Neumann and Weiss observe that, with respect to traditional sectors, high-tech sectors are subject to a higher rate of technological innovation. Since high-tech sectors use a highly specialized workforce - that is a workforce composed by engineers, computer scientists, biologists, etc - at any given time, we may reasonably expect a higher rate of obsolescence for the most specialized workers, which are typically the most educated ones. By assuming that the degree of knowledge specialization increases with the grade of education, we may conclude that: "at the individual level, the knowledge obsolescence increases as specialization increases" [McPherson and Winston (1983)].

2. The second cause is known as "technical depreciation" and concerns the fact that the speed of technological change has accelerated over time. As stressed in de Grip (2004), in a context in which innovation accelerates over time, we must expect that even the obsolescence rate of knowledge accelerates continuously. This view is largely supported by the experience of the past several decades, where technological changes have produced an increasing demand of skills by the firms. This happens independently of the level of education of a worker. Today an engineer, with respect to thirty years ago, experiences a higher rate of knowledge obsolescence, hence knowledge becomes obsolete more quickly in any production sector. For this reason, it is reasonable to assume that an increasing speed of innovation leads to an increasing depreciation rate of human capital.

[7] Differently, MRW (1992) consider a constant obsolescence rate of human capital.

Even if the above mentioned causes of human capital obsolescence can operate separately, we expect to observe both phenomena simultaneously. According to Rosen (1976), these two causes of human capital depreciation are indistinguishable. However, Neuman and Weiss (1995) argue that since the *vintage effect* is not the same in all sectors while technical depreciation is, it is possible to identify both causes by comparing data from low and high technology sectors. Consistently with this point of view and with our future assumption of a constant exogenous rate of technological progress, we will consider only the *vintage effect*.

Therefore, by considering Equation (2), we can write the following human capital accumulation function:

$$\frac{dh_{i,t}}{de_{i,t}} = Bh(e_{i,t}) - \mu(h(e_{i,t}))h(e_{i,t}), \quad \frac{\partial \mu}{\partial h} > 0, \tag{3}$$

where $\frac{dh_{i,t}}{de_{i,t}}$ is the human capital variation with respect to $e_{i,t}$, $B > 0$ is a measure of gross productivity of the education sector, and $\mu(.)$ is a depreciation coefficient used to capture the loss of human capital. In Equation (3), the term $Bh_{i,t}$ represents the gross creation of human capital due to the acquisition of formal education, while the term $\mu(h_{i,t})h_{i,t}$ represents the depreciation of human capital due to obsolescence.

By considering Equations (1) and (2) and assuming a linear relationship between the stock of human capital and its depreciation rate, that is $\mu(h_{i,t}) = \sigma h_{i,t}$, we have:

$$\frac{dh_{i,t}}{de_{i,t}} = Bh(e_{i,t}) - \sigma h(e_{i,t})^2 \tag{4}$$

where σ is a positive parameter that represents the unitary variation of the depreciation coefficient of human capital. We assume that $B > \sigma$, that is, the productivity of education sector is greater than the unitary variation of the depreciation coefficient due to the *vintage effect*. This hypothesis ensures that an additional investment in education will always increase the individual stock of human capital.

To study how our economic system produces human capital, we have to solve Equation (4).[8] Without loss of generality, we can take an initial stock of human capital $h(0) = 1$, thus we obtain:

$$h_{i,t} = \frac{B \exp[Be_{i,t}]}{B - \sigma + \sigma \exp[Be_{i,t}]} \tag{5}$$

[8]Equation (4) is an autonomous differential equation also known as Verhulst's equation [from Verhulst (1845)]. In particular, here, as independent variable we have used a simple linear transformation of time ($e_{i,t} = \alpha_i t$). As well shown by Richards (1959), this class of equations can be solved by the method of separation of variables.

Equation (5) represents our human capital production function, which depends on the individual choices about the schooling level. By considering the net flow of knowledge, some nonlinearities appear in the process of human capital accumulation. In fact, according to (5), the individual level of human capital will not be directly proportional to the time invested in education by agent i. In particular, here we have obtained a *logistic production function of human capital.* Note that, in this model, the maximum level of human capital that an individual can accumulate ($H_{\max} = \frac{B}{\sigma}$) does not depend on any biological consideration on the capacity of human brain, but it depends positively on the productivity of education sector and negatively on the unitary variation of the depreciation coefficient of human capital. By assuming $\sigma = 0.017$ and $B = 1$, in Figure 1 we provide a graphycal representation of Equation (5).

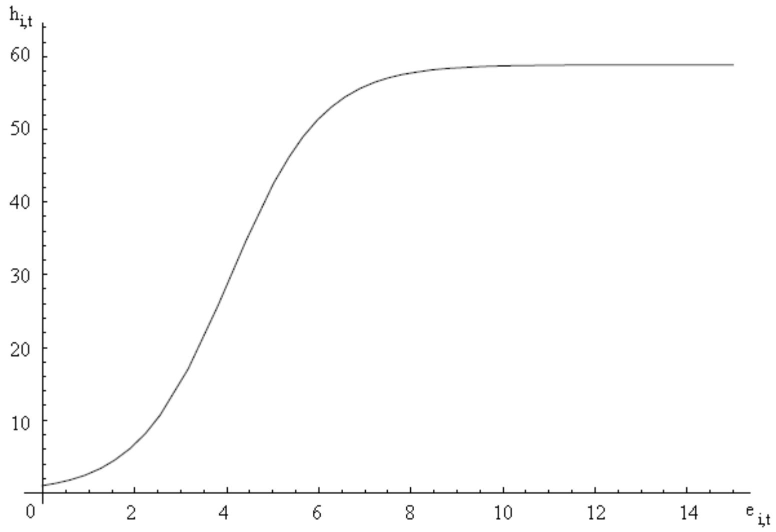

Figure 1

Figure 1 shows how returns to education in the production of human capital are increasing for low levels of education and decreasing for high level of education. In particular, if human capital depreciation increases of about two percentage points for each additional unit of human capital, the production function of human capital starts to show decreasing returns to education after about 4.5 years of schooling. In Section 3, we will see that the assumption of $\sigma = 0.017$ is consistent with empirical evidence.

We can interpret Figure 1 also in another way. In fact, Figure 1 states that the effort (in terms of time invested in education) necessary to acquire an additional unit of human capital initially decreases with the stock of human capital already accumulated by an individual and subsequently increases. This means that an additional year of schooling has different effects if it is attended by an individual with a low level of human capital or by an individual with a high level of human capital.[9]

By using a neoclassical framework, in the next section, we study the consequences of a logistic process of human capital accumulation on output growth.

2.2 A modified Solow growth model

2.2.1 Production function

In this model, we use a Cobb-Douglas production function with constant returns to scale:

$$Y_t = K_t^{\beta}(uH_tA_t)^{1-\beta}, \quad \beta \in [0,1] \ . \tag{6}$$

where K_t is the total stock of physical capital, H_t is the total stock of human capital, and A_t is a technology parameter. In Equation (6), human capital and technological progress enter multiplicatively. That is, technological progress is *human capital-augmenting*. As we will see, this hypothesis will allow us to write the production function in terms of *units per effective-labor* by making the analysis much simpler.

Assuming that there are not external effects in the aggregation of human capital, that is, the total stock of human capital is simply the sum of individual stocks of human capital, we can aggregate individual capacities as follows:

$$H_t = \int_{i=0}^{L_t} h_{i,t}di = L_t\overline{h}_t \tag{7}$$

where $\overline{h}_t$ is the average level of human capital at time t. Equation (7) states that, at the aggregate level, what matters is the average level of human capital. At the same time, according to (5), the average level of human capital will depend on the average level of education.

Therefore, by putting Equation (5) into Equation (7) we have:

$$H_t = L_t\frac{B\exp[B\overline{e}_t]}{B-\sigma+\sigma\exp[B\overline{e}_t]}. \tag{8}$$

[9] In this work, we do not consider the implications of our model in terms of welfare analysis, nevertheless this consideration may represent an interesting starting point for future studies.

where $\overline{e}_t$ represents the average level of education in our economic system at time t.

Finally, by considering Equations (6) and (8), we can rewrite Equation (6) in terms of quantities *per unit of effective-labor* (uLA) as follows:

$$y_t = k_t{}^{\beta} \left(\frac{B \exp[B\overline{e}_t]}{B - \sigma + \sigma \exp[B\overline{e}_t]} \right)^{1-\beta} \tag{9}$$

where k_t and y_t are respectively the amount of physical capital and output per unit of effective-labor, that is, $k_t \equiv \frac{K_t}{uL_tA_t}$ and $y_t \equiv \frac{Y_t}{uL_tA_t}$. In the production function described by (9), education is not a proxy variable for human capital as usual but enters as input in the production of human capital. In the empirical part of this work, we will compare this formulation with a more standard Solow model augmented by the presence of human capital.

In the next section, we examine the dynamic behavior of the inputs into production. Since this is a fairly standard analysis, we will especially emphasize the main differences of our model with respect to the standard version of the Solow model.

2.2.2 Dynamics of the inputs into production

First, as in the exogenous growth theory, we assume that technology and labor grow at the exogenous rates g and n, respectively. Formally, we will have that:

$$\dot{A}_t = gA_t \tag{10}$$

and

$$\dot{L}_t = nL_t. \tag{11}$$

Second, as in the Solow model, output is divided between consumption and investment. By considering the physical capital depreciation from one period to the other, we can write the accumulation function of physical capital as follows:

$$\dot{K}_t = I_t - \delta K_t \tag{12}$$

or

$$\dot{K}_t = sY_t - \delta K_t \tag{13}$$

where I_t is the gross investment at period t, s is the exogenous saving rate; and δ is the exogenous depreciation rate of physical capital.

2.2.3 Dynamic solutions

Now, we must study the dynamic behavior of physical capital; we can consider the whole dynamics of our economic system through the classic accumulation function of physical capital:

$$\dot{k}_t = sy_t - (\delta + g + n)k_t. \tag{14}$$

By substituting y_t with (9):

$$\dot{k}_t = s[{k_t}^{\beta} h(\overline{e}_t)^{1-\beta}] - (\delta + g + n)k_t. \tag{15}$$

Expression (15) is similar to the Solowian dynamic equation of capital per labor services, except for the presence of human capital. Given Equation (15), we can find the steady-state level of physical capital per capita (k^*) simply equalling $\dot{k}_t$ to zero and solving for k_t :

$$k^* : \quad s(k^*)^{\beta} h(\overline{e}_t)^{1-\beta} = (\delta + g + n)k^* \tag{16}$$

Once we have found k^*, we can put it into Equation (9) in order to obtain the steady-state level of output given a certain average level of education.

Until now, in the equations of the model, physical and human capital have always been considered separately. Nevertheless, some macroeconomic studies suggest that output growth (and consequently physical capital accumulation) can influence the average level of education.

For example, Bils and Klenow (2000) find that the level of education registered in a country depends positively on its GDP growth. Bils and Klenow investigate the presence of a reverse causality between human capital and growth. In contrast with previous studies such as Barro (1991), Benhabib and Spiegel (1994) and Barro and Sala-i-Martin (1995), they conclude that the causality direction is from economic growth to schooling and not the opposite. In a more recent work, Glewwe and Jacoby (2004) find a positive relationship between the individuals' investment in education and their levels of wealth.

Therefore, in line with these studies, we assume that, at the aggregate level, there exists a positive relationship between the physical capital per capita of a country and the average level of education of its population. The introduction of these complementarities in the accumulation of physical and human capital will allow us to explain the level of output only in terms of physical capital per unit of effective-labor, as in the Solow model. In this way, we will obtain a growth model in which nonlinear dynamics in the economic growth path may appear.

The assumption of a positive relationship between the physical capital of a country and the average level of education of its population can be formalized as follows:

$$\overline{e}_t = k_t^b, \quad b > 0. \tag{17}$$

Equation (17) states that in a rich country, individuals invest more time in education than those in a poor country. That is, the average stock of human capital depends positively on the stock of physical capital per unit of effective-labor and on the constant elasticity coefficient of e with respect to k, that is, b.

By combining Expressions (5), (9) and (17) we can obtain a more specific formulation for y_t in which output depends only on physical capital (and therefore, we can obtain the dynamics of y_t from the dynamics of k_t):

$$y_t = k_t^{\beta} \left(\frac{B \exp[\underline{B} b k_t]}{B - \sigma + \sigma \exp[\underline{B} b k_t]} \right)^{1-\beta} \tag{18}$$

where $\underline{B} \equiv B^{\frac{1}{b}}$. Now, the steady-state solutions of our augmented Solow model can be found as follows:

$$k^* : \quad s(k_t^*)^{\beta} \left(\frac{B \exp[\underline{B} b k_t^*]}{B - \sigma + \sigma \exp[\underline{B} b k_t^*]} \right)^{1-\beta} = (\delta + g + n) k_t^* \tag{19}$$

In the Solow model, the saving function, sy_t, is always convex with respect to the origin of axes, and this implies that if a steady-state level of k_t exists this level will be unique. While in our model, given the presence of initial increasing returns to k_t in the accumulation of human capital, the saving function is initially concave and subsequently becomes convex. This fact generates the possibility to observe multiple steady-state levels of k_t. Figure 2 provides a graphical representation of a situation in which multiple equilibria appear.[10] In particular, the low equilibrium (k_L) and the high equilibrium (k_H) are stable, while the middle equilibrium (k_M) is unstable.

[10] As well discussed by Galor (1996), a nonmonotonic saving function is a necessary, but not sufficient, condition for the existence of multiple equilibria. For example, if s is sufficiently high (or $n + g + \delta$ is sufficiently low) the function $sy_t(k_t)$ will match the function $(n + g + \delta)k_t$ only one time.

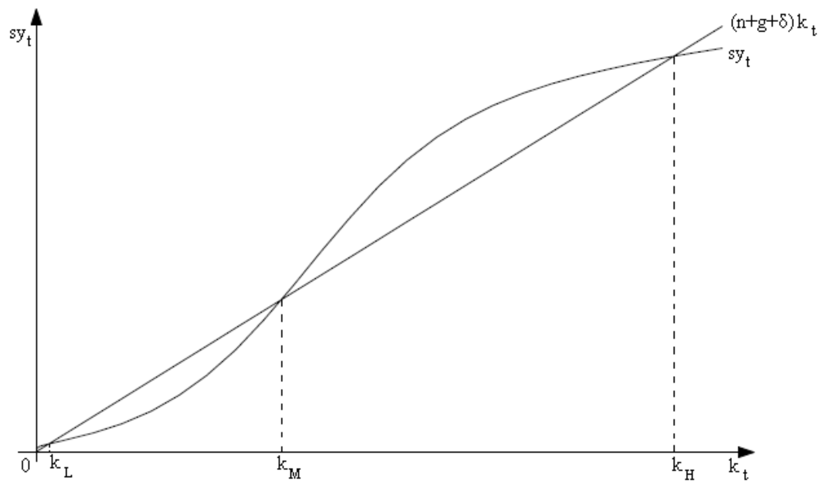

Figure 2

Let us examine the dynamics of physical capital implied by Figure 2. Assuming an initial level of physical capital k_0 such that $0 < k_0 < k_L$ (or $k_M < k_0 < k_H$), Figure 2 states that total saving will exceed the minimum level of saving necessary to hold the level of physical capital per unit of effective-labor constant, therefore, we will observe an increase in the level of k_t. This increase in k_t will rise the level of output either directly as input for the production of y_t or indirectly causing an increase in the level of education and consequently of human capital. In this situation, k continues to rise until it reaches the value k_L (or k_H), at this level physical capital will remain constant.

On the contrary, if $k_L < k_0 < k_M$ (or $k_H < k_0 < +\infty$), total saving will be lower than the minimum level of saving necessary to hold the level of physical capital per unit of effective-labor constant, therefore, we will observe a decrease in the level of k_t until it will reach the value k_L (or k_H). Therefore, we can conclude that the convergence of k_t towards the value k_L or the value k_H depends on the initial value of physical capital per unit of effective-labor, k_0.

Figure 2 also suggests that, under the assumption of a constant saving rate, the returns to physical capital in the production of output are increasing for low levels of k_t and decreasing for high levels of k_t. Moreover, given Equation (9) and the assumption represented by Equation (17), the same relationship should be observed between the output growth and the average years of schooling. In the empirical part of this paper, we test if these theoretical conclusions are supported by evidence.

Before proceeding, note that we can write Equation (19) and consequently the steady-state solutions of the model as follows:

$$k_t^*: \quad F(k_t^*) = \delta + g + n. \tag{19'}$$

where $F(k_t) \equiv s k_t^{\beta-1} \left(\frac{B \exp[\underline{B} b k_t]}{B-\sigma+\sigma \exp[\underline{B} b k_t]} \right)^{1-\beta}$ and k_t^* is still the steady-state solution of (19).

As is well known, in the Solow model the derivative of the equivalent of $F(k_t)$ is negative for all k, this implies that, if an interior steady-state equilibrium exists, this equilibrium will be unique and condition (19') will be fulfilled. On the other hand, multiplicity requires a nonmonotonic form for F, that is, F' must be positive for some values of k. Multiple equilibria are the result of multiple intersections between F and the constant value $\delta + g + n$. After some algebra (see Appendix C), we find that this derivative may be positive for some values of k. In particular, F' will be positive if and only if:

$$\frac{b\underline{B}(B-\sigma)}{B-\sigma+\sigma \exp[\underline{B} b k_t]} > \frac{1}{k_t} \tag{20}$$

Equation (20) states that, given certain values of parameters, we cannot exclude the possibility that several stable steady-state equilibria exist. By using the condition expressed by (19'), in the next section, we provide a numerical example in which we show that, under plausible values of parameters, the model presented in this section can explain the existence of multiple equilibria.

3 Numerical Example and Consistency

Here, we provide a numerical example to illustrate the main implications of our model. We set up the exogenous parameters of the model by using those observed for the European Union (EU-15) in 2001.[11] There, the depreciation coefficient of physical capital was not available for all countries; thus, we have taken the depreciation coefficient observed by MRW (1992). Table 1 contains the parameter values used to draw Figure 3.

In Table 1, the value of σ refers to the one estimated by Arrazola et al. (2005) for Spain, while the parameter b has been estimated by using the data set provided by Levine et al. (2000) and described in the next section.[12] Finally, since there

[11] Source Eurostat (2004).

[12] In order to obtain b, we have regressed the logarithm of the average years of education, e_t, on the logarithm of the current stock of physical capital per capita, k_t. That is, by using the OLS method, we have estimated the following log-linear transformation of Equation (17): $\ln(e_t) = b\ln(k_t)$.

Table 1: Parameters used in the numerical example	
Parameter	Value
s	0.127
b*	0.504
β	0.250
σ**	0.017
δ***	0.030
g	0.028
n	0.014
* from Levine et al. (2000)	
** from Arrazola et al. (2005)	
*** from MRW (1992)	

Figure 1:

is not only one way to measure the education system productivity, and these measures are relative indices rather than absolute indicators, we have decided to assume the same productivity for all education systems, and in particular $B = 1$.

Figure 3 shows that, under plausible values of the exogenous parameters, the function $F(k_t)$ is nonmonotonic. Nevertheless in Figure 3, since we have used parameters from developed countries, $F(k_t)$ equals the constant value $n + g + \delta$ in only one point. This means that for countries of EU-15 we have a unique steady-state equilibrium (k^*).

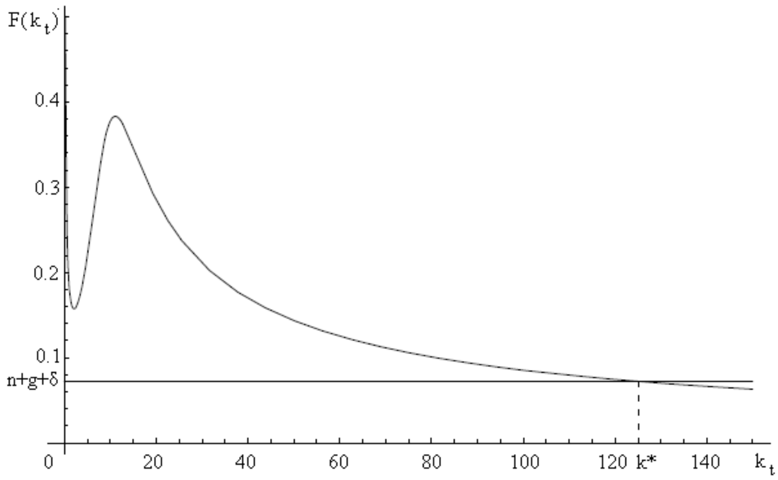

Figure 3

Figure 4 shows what happens to F when the elasticity coefficient of e with respect to k, that is b, changes. In particular, if the value of b is 0.10, the function F will match the constant value $n+g+\delta$ in three different points (k_L, k_M, k_H). These points are three different steady-state equilibria, two of which are stable (k_L, k_H). In this situation, as discussed in the previous section, countries with a stock of physical capital per capita lower than k_M will converge to the equilibrium characterized by a low level of physical capital per capita, k_L: that is, these countries fall in a *poverty trap.* At the same time, countries with a stock of physical capital higher than k_M will converge to the highest equilibrium, k_H. Figure 4 also shows how a sufficiently large increase in b ($b = 0.50$) will shift the function F up enough to eliminate the lower steady-state. This means that a highly educated society, that is a society in which individuals spend a large part of their time to study, will converge to a high level of physical capital per capita. In fact, above a certain level of b, the lower and the middle equilibria disappear, and we will observe only one (high) equilibria. This result supports the idea that, policies devoted to increase the average years of schooling in less developed countries can help these countries to escape from a situation of poverty trap.[13]

[13] In Figure 4 an increase in b has the same effect that an increase in the saving rate, s, has in Galor (1996). In both cases $F(k)$ shifts up and only one equilibrium remains.

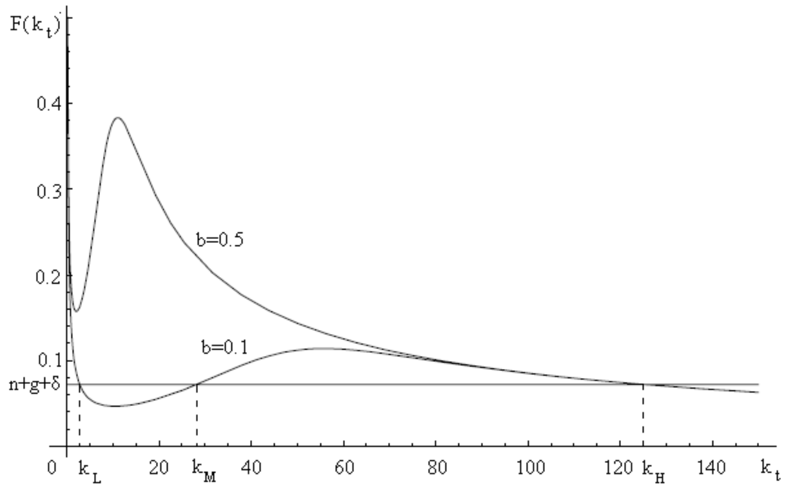

Figure 4

Figure 5 shows that if β rises our graph becomes smoother. This is due to the fact that we are keeping the hypothesis of constant returns to scale in production. That is, if β increases, the elasticity coefficient of human capital $(1-\beta)$ decreases and the nonlinear effect of education becomes relatively less important. Note that when β increases multiple equilibria may appear, disappear or remain. In particular, in Figure 5 an increase in β implies only a higher steady-state level of k.

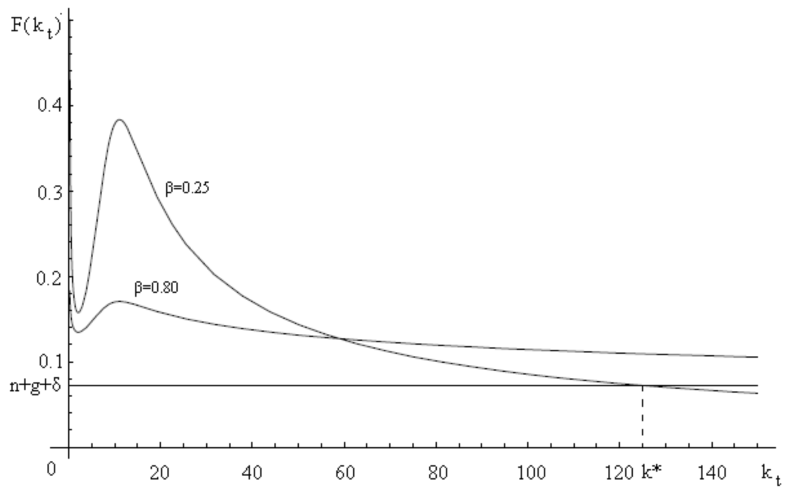

Figure 5

Finally, let us discuss the effect of a change in the value of σ. Figure 6 shows that multiple equilibria tend to disappear when the unitary depreciation of human capital due to vintage effect rises. In fact, an increase of σ slows down the process of human capital accumulation which causes the existence of nonlinear dynamics in our model. Therefore, if σ increases enough, we will observe only one equilibrium as in the traditional Solow model. However, in this situation, given the small contribution of human capital to the production of output, the unique steady-state equilibrium will be lower than the highest equilibrium observed in a situation with multiple equilibria. Symmetrically, when σ decreases, the accumulation of human capital becomes easier and nonlinearities more pronounced. This means that, given the assumption of an increasing depreciation coefficient of human capital (due to vintage effect), the dynamics of output growth tends to be nonlinear, and these nonlinearities are more important when σ is smaller.

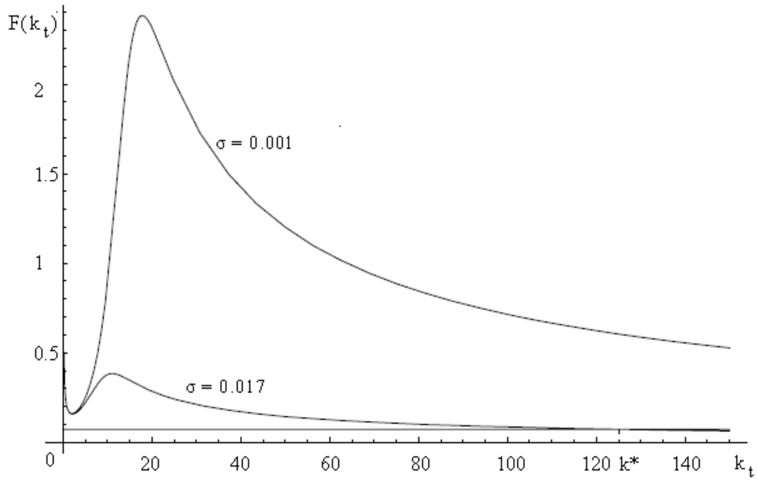

Figure 6

Since a numerical example can display only one possible outcome for a model, we should view all conclusions as possibilities rather than as general results. Nevertheless, we can conclude that our model is consistent with different possible settings of principal parameters.

4 Empirical Analysis

In this section, we want to verify the validity of our theoretical conclusions. That is, we test the existence of a nonlinear relationship between education and GDP growth and between physical capital and GDP growth. In fact, according to our theoretical model, we should observe an inverted-U shaped relationship between education and GDP growth and between physical capital and GDP growth. To do this, we estimate different econometric specifications. In particular, we compare the Solow growth model (augmented by the presence of human capital) with two econometric specifications consistent with the two nonlinear production functions represented by (9) and (18). To investigate the presence of nonlinearities in the production of output, we will use a semiparametric specification already used by Liu and Stengos (1999).

4.1 Data and Variables

Data come from Levine, Loayza, and Beck (2000). Their sample consists of 78 countries with data for the period 1960-1995. In order to eliminate the business cycle effects, data have been collected in groups of five-year averages.[14] The dataset contains 24 quantitative variables and 4 dummies. Table 2 provides a description of variables employed in our regressions.

Table 2: Description of variables

g_y	GDP growth rate
g_k	Growth rate of physical capital
g_e	Education growth rate
g_{pop}	Population growth rate
$D70$	Dummy variable for years 1970-79
$D80$	Dummy variable for years 1980-89

Variables are expressed in terms of 5 years average.

In order to measure human capital, we should consider different dimensions: years of schooling, quality of schooling, work experience, general and specific skills. Unfortunately, with respect to the number of years of schooling, the other dimensions are not so easy to quantify. A standard proxy variable, used to measure the human capital level, is the enrollment rate in secondary education. In fact, according to most authors, secondary education develops the basic education obtained

[14] In Appendix B we have reported the country list.

in primary school, and allows for future learning and human capital growth. That is, secondary school would provide a more specialized education, and then a more skilled workforce. In contrast to this view, we think that the division between primary and secondary education is arbitrary, therefore we cannot discriminate when a subject starts to develop his or her human capital. That is to say that each step, during the process of human capital accumulation, is necessary for the subsequent step. For example, when a future writer learns to read he or she is acquiring a very specific knowledge for his or her future human capital. Therefore, we prefer to measure the impact of education on growth by using the average years of schooling, without distinction between primary and secondary education.

4.2 Econometric Specification

In the Solow model, the aggregate output is produced by a Cobb-Douglas production function. This assumption implies a log-linear relationship between aggregate output and the inputs used to produce it, that is, a linear relationship between the growth rate of output and the growth rate of inputs. Therefore, a fairly standard specification for the GDP growth equation is:

$$g_{yt} = \beta_1 g_{k_t} + \beta_2 g_{h_t} + \gamma X_t^T + \varepsilon_t. \tag{21}$$

where: g_{y_t} is the GDP growth rate at time t; g_{k_t} is the growth rate of physical capital per capita; g_{h_t} is the growth rate of human capital per capita; X_t is a vector of control variables expressed in terms of growth rates in which the first element is assumed to be a vector of 1s, so that γ_1 will be the intercept of our empirical models; β_1, and β_2 are, respectively, the coefficients of g_{k_t}, and g_{h_t}; γ is a vector of coefficients; and ε_t is a white noise error. In Equation (21), following the traditional literature, we have assumed a production function of human capital equal to $h = e$. To avoid the problem of heteroscedasticity for cross-section data, we have estimated model (21) by a traditional GLS method.[15]

In order to verify the hypothesis of a nonlinear contribution of education to economic growth, as suggested by Equation (9), we estimate a more general production function in which the effect of education on GDP growth is unknown a priori. In this way, the contribution of education to the production can be estimated without restrictions. In particular, we use a semiparametric specification in which education enters in the nonparametric part of a PLR (Partial Linear Regression) model. The semiparametric regression equation used can be written as follows:

[15] GLS and PLR estimates are obtained by using R programs provided by Abbey (1988) and Wood (2006), respectively.

$$g_{y_t} = \beta_1 g_{k_t} + \Phi(\overline{e}_t) + \gamma X_t^T + \varepsilon_t. \tag{22}$$

where $\Phi(\overline{e}_t) \equiv \frac{d \ln \phi(\overline{e}_t)}{dt}$ and $\phi(\overline{e})$ is the unknown function of Equation (2), which has been estimated by using a normal kernel with bandwidth obtained by cross-validation. According to Equation (9), returns to education in the production of output (via human capital accumulation) are increasing for low levels of education and decreasing for high level of education. Therefore, by looking at the shape of $\Phi(\overline{e}_t)$, we expect to observe a positive relationship between education and GDP growth for relatively low levels of education and a negative relationship for relatively high levels of education.[16]

Since we are interested to test the validity of (18), we also estimate an econometric model in which physical capital and not education enters in the nonparametric part of a PLR model. In particular, we estimate the following model:

$$g_{y_t} = \Psi(k_t) + \gamma X_t^T + \varepsilon_t. \tag{23}$$

where $\Psi(.)$ is an unknown function, which has been estimated by using a normal kernel with bandwidth obtained by cross-validation.

Finally, we can say that, given the assumption contained into (17), $\Phi(.)$ and $\Psi(.)$ should have the same shape. In particular, as the shape of $\Phi(.)$ should be consistent with the hypothesis of an inverted-U relationship between education and GDP growth, the shape of $\Psi(.)$ should be consistent with the hypothesis of an inverted-U relationship between k_t and GDP growth.

4.3 Empirical Results

Table 3 contains our estimates for equations (21), (22), and (23). The coefficients on control variables enter with the expected signs in all of three regressions, and they are statistically significant at 1%, except for the dummy variable for period 1970-1980.

[16] Note that, econometric models are formilized in terms of GDP growth rates instead of GDP growth, nevertheless, the interpretation of our empirical results will not present any particular difficulty.

Table 3: Cross-Country Regressions on GDP growth rate (1960-95)			
	GLS (1)	PLR (2)	PLR (3)
Intercept	0.0303 (4.249)**	0.0331 (4.608)**	0.0339 (4.330)**
$D70$	-0.0038 (1.425)	-0.0044 (1.629)	-0.0036 (1.301)
$D80$	-0.0175 (6.683)**	-0.0180 (6.794)**	-0.0171 (6.457)**
g_{pop}	-0.0049 (3.986)**	-0.0043 (3.475)**	-0.0041 (2.430)**
g_k	0.2454 (7.705)**	0.2345 (7.354)**	
g_e	-0.0054 (2.508)**		
Obs	499	499	499
Adj-R^2	0.308	0.315	0.309
Log-Lik	1154.65	1163.17	1157.69
GCV		0.00059	0.00060
d.f.	8	13.51	11.43

Parentheses contain the values of t-statistics. Significance levels: ** (1%), * (5%)

Column (1) reports the estimated results for Equation (21). There, the coefficient on growth rate of physical capital per capita is positive (0.2454) and statistically significant at 1%. While, the coefficient on education growth rate is negative (-0.0054), and statistically significant at 1%; education growth seems to have a negative effect on GDP growth. This result is rather counterintuitive; this means that either education is a damaging factor to produce output or education is a source of nonlinearities in the process of economic growth. As we will see, this second hypothesis is confirmed by the next regression.

Column (2) contains the estimated parameters for the semiparametric specification given by Equation (22), in which education enters in an a priori unknown way. This model explains data in a better way with respect to the previous regression. The log-likelihood ratio is the highest, and the Generalized Cross Validation (GCV) score is smaller than the GCV score calculated for the PLR model in Column (3).[17] Figure 7 presents the estimated function $\Phi(e_t)$. By comparing this function with the confidence interval band of the simple linear specification, we can conclude that a nonlinear relationship between education and GDP growth rate emerges markedly. The function $\Phi(e_t)$ falls within the band of the linear specification only for two small intervals of education attainment. Figure 7 states that education and the GDP growth rate are positively related for relatively low levels of education and negatively related for relatively high levels of education. That is, as suggested by our theoretical model, an increase in the level of schooling seems to be more profitable for countries with low levels of education than for countries with high levels of education. This result confirms the previous findings provided by Liu and Stengos (1999) and Kalaitzidakis et al. (2001) concerning the existence of a nonlinear relationship between education and GDP growth. Nevertheless, with respect to the previous works, now, we have a theoretical model which explains the existence of increasing returns to education for low levels of schooling and decreasing returns to education for high level of schooling.

[17] GCV criterion is a technique used to guide the selection of optimal parameters in smoothing splines and related regularization problems. The optimal combination of parameters is, typically, the one that produces a minimum GCV score. The GCV score function is defined as: $GCV(\alpha, \rho) = (\sum_{i=1}^{n} \frac{e_i^2}{n})/(1 - \frac{m}{n})^2$ where e is the vector of error terms, m is the number of parameters, n is the number of data points, $\alpha = \frac{K}{n}$ (K is the number of sub-samples) and ρ is the order of polynomial used to fit the data into the sub-samples.

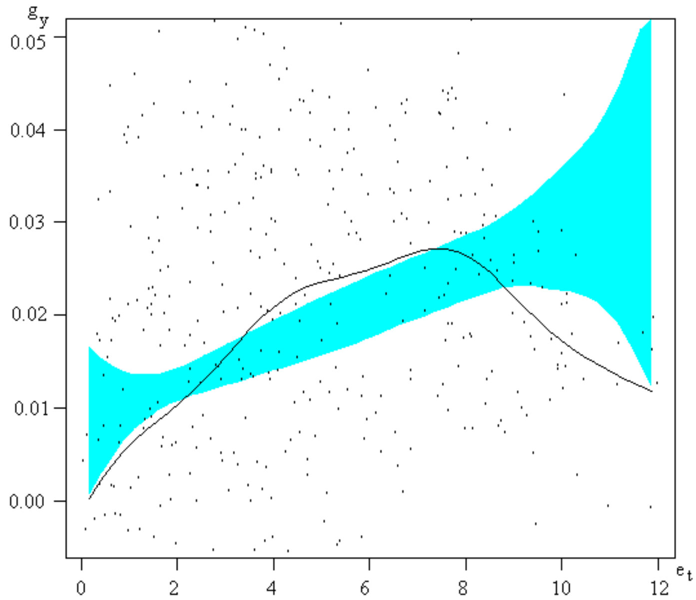

Figure 7

In Column (3), we have reported the estimates for the parametric part of Equation (23), in which physical capital is related to economic growth in an a priori unknown way. This PLR specification fits data better than Equation (21); nevertheless, the best fit is provided by Equation (22). Figure 8 presents, graphically, the nonlinear component of Equation (23), that is $\Psi(k_t)$. A nonlinear relationship emerges also between the current stock of physical capital and the GDP growth rate. In particular, this relationship is increasing for low levels of physical capital per capita and decreasing for high levels of physical capital per capita. This evidence contrasts with the prediction of the Solow model concerning the inverse relationship between the stock of physical capital per capita of a country and its GDP growth rate.[18] Whereas the shape of $\Psi(k_t)$ is consistent with the production function described by Equation (18).

[18] In the Solow growth model, when an economy is below its steady-state value of physical capital per unit of effective-labor, the marginal product of physical capital is higher than in the steady-state equilibrium. Therefore, a given investment in physical capital implies a relatively high output growth. In this way, also physical capital grows but, given the presence of decreasing returns to k_t, the capital-output ratio rises and the marginal product of capital decreases. Therefore, the growth rates of output and physical capital slow down.

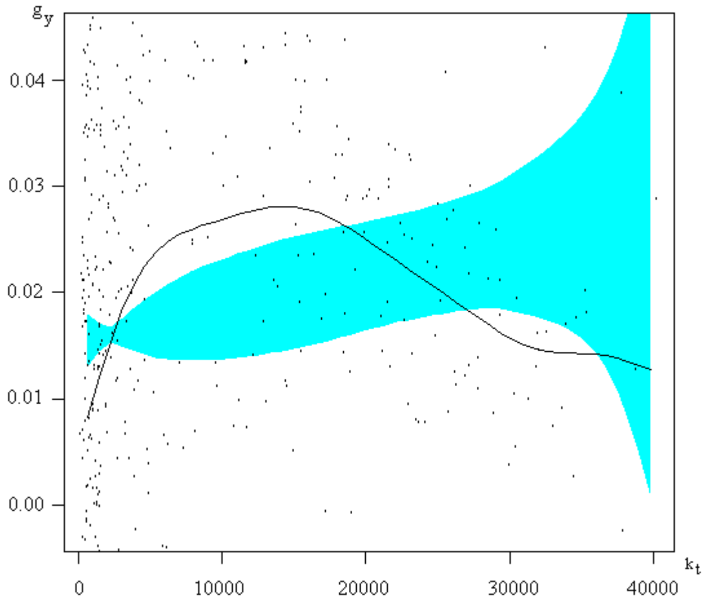

Figure 8

To support the assumption contained into Equation (17), in Figure 9 we have reported the results of a nonparametric estimate of the relationship between the stock of physical capital and the average years of schooling. There, a new evidence emerges, that is the existence of a positive relationship between the average level of education and the current stock of physical capital per capita. Thus, the accumulation of human capital is positively related to the accumulation of physical capital, this means that there are some complementarities in the accumulation of both kinds of assets. In fact, as shown by our model, if the accumulation of physical capital induces individuals to devote a larger fraction of their time to education, and if education is a source of nonlinearities in the accumulation process of human capital, physical capital accumulation becomes the primary cause of these nonlinearities.

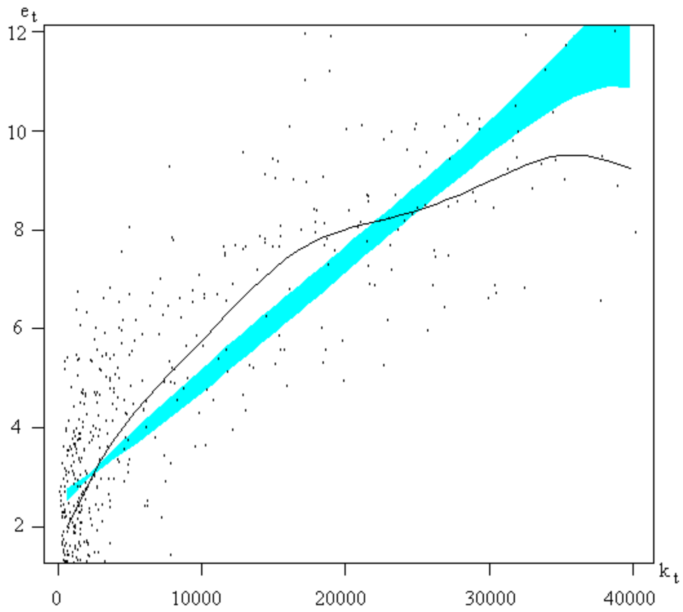

Figure 9

In Figure 10, we show how the empirical results obtained in this section can be related each others. In particular, in this figure, we compare the shape of $\Phi(e)$ with the shape of $\Psi(k)$. As we can see, these two functions have the same form, and the link between these two functions seems to be the assumption contained into Equation (17). This suggests that our growth model - in which the accumulation of human capital depends nonlinearly on the investment in education (in terms of average years of schooling), and the investment in education depends on the stock of physical capital per capita - is supported by empirical evidence. In other words, empirical results support the idea that the accumulation process of human capital, and consequently the GDP growth path of an economy, may be nonlinear. According to Figure 10, a country with a relative low level of physical capital will also have a relative low level of education and thus a low level of human capital. In this situation, in contrast with the prediction of the Solow model, the GDP growth rate of this country may be lower than the GDP growth rate of a country with a higher stock of physical capital.[19] Moreover, in line with the *club convergence*

[19] Consistently with the theoretical model provided in Section 2, we have interpreted our empirical results by assuming that education, and not human capital, is a source of nonlinearities in the process of economic growth. In fact in our model, human capital enters in a Cobb-Douglas production function with constant returns to scale, while time invested in education causes a nonlinear accumulation process of human capital.

hypothesis, Figure 10 shows that the best growth performances are observed in those countries whose stocks of human and physical capital are neither too high nor too low.

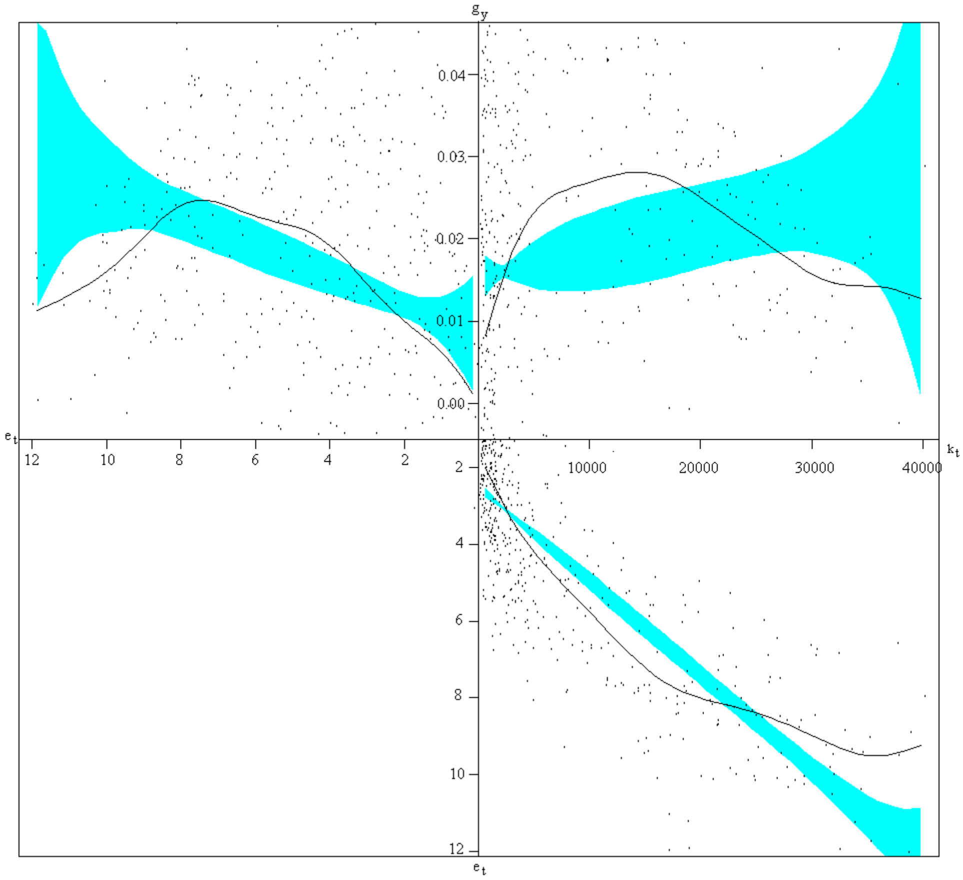

Figure 10

5 Conclusions

In this paper we have studied the effect of education on economic growth. In particular, we have shown theoretically and empirically that education can have a nonlinear effect on the process of economic growth. By assuming that higher educated workers are more affected by depreciation of human capital than lower educated workers, in Section 2 we have provided a theoretical model, which explains why the returns to scale in producing human capital through education initially increase and later decrease.

Subsequently, we have used this result to modify a Solow model augmented by the presence of human capital. There, we have shown that a positive relationship between physical capital and education leads to a situation in which multiple equilibria may arise in the GDP growth path. Through a numerical example, Section 3 shows that, under plausible values of the exogenous parameters, our model may generate the expected nonlinearities and consequently the possibility of multiple equilibria in the economic growth path. Moreover, we have studied the effects of parameter variations on these equilibria. In particular, we have seen how investing in education is one way for poor countries to exit from an equilibrium characterized by low levels of physical capital per capita.

In the last part of the paper, we have investigated the empirical relationship between education and growth. By comparing traditional parametric regressions with a semiparametric regression, we found evidence in favor of a nonlinear relationship between education and growth. In particular, we have seen that for low levels of education, an increasing relationship emerges between years of schooling and GDP growth rate, while for high levels of education, the same relationship is decreasing. Since education and physical capital are positively related, we have shown that a nonlinear relationship emerges also between physical capital and GDP growth.

In conclusion, a PLR model, in which education enters into the nonparametric part of the model, fits data better than traditional regressions. Perhaps education is not the only cause for a nonlinear economic growth. Nevertheless, this paper suggests that the accumulation process of human capital may be one of the determinants of these nonlinearities.

A The Nonmonotonic Behavior of $F(k)$

If $F(k_t)$ is monotone, it will match the constant value $n+g+\delta$ only once. Therefore, the nonmonotonic form of $F(k_t)$ is a necessary condition for the existence of multiple equilibria. To show that $F(k_t)$ can be nonmonotonic for some value of its parameters, we must study the sign of its derivative with respect to k_t.

We can write $F(k_t)$ as follows:

$$F(k_t) \equiv sf(k_t)g(k_t) \tag{A.1}$$

where $f(k_t) \equiv k_t^{\beta-1}$ and $g(k_t) \equiv (\frac{B\exp[\underline{B}bk_t]}{B-\sigma+\sigma\exp[\underline{B}bk_t]})^{1-\beta}$.

Thus, the derivative of $F(k_t)$ with respect to k_t is:

$$F'(k_t) = s[f'(k_t)g(k_t) + f(k_t)g'(k_t)] \tag{A.2}$$

Since s is a positive constant, we can say that $F'(k_t) \gtreqless 0$ if and only if:

$$\frac{g'(k_t)}{g(k_t)} \gtreqless -\frac{f'(k_t)}{f(k_t)} \tag{A.3}$$

or

$$\frac{b\underline{B}(B-\sigma)}{B-\sigma+\sigma\exp[\underline{B}bk_t]} \gtreqless \frac{1}{k_t} \tag{A.4}$$

If $B > \sigma$, we have:

$$\lim_{k_t\to 0^+} \frac{g'(k_t)}{g(k_t)} < \lim_{k_t\to 0^+} -\frac{f'(k_t)}{f(k_t)} \tag{A.5}$$

Thus, $F'(k_t)$ will be negative as $k_t \to 0^+$.

Now, we must state the conditions under which $F'(k_t)$ is positive, i.e.:

$$\frac{b\underline{B}(B-\sigma)}{B-\sigma+\sigma\exp[\underline{B}bk_t]} > \frac{1}{k_t} \tag{A.6}$$

Given (A.5) and (A.6), we can conclude that, for some configuration of parameters, $F(.)$ is a nonmonotone function.

B The Country-List

Table D.1: The Country-List		
Algeria	Guatemala	Pakistan
Argentina	Guyana	Panama
Australia	Haiti	Papua New Guinea
Austria	Honduras	Paraguay
Belgium	India	Peru
Bolivia	Indonesia	Philippines
Brazil	Iran, Islamic Republic of	Portugal
Cameroon	Ireland	Rwanda
Canada	Israel	Senegal
Central African Republic	Italy	Sierra Leone
Chile	Jamaica	South Africa
Colombia	Japan	Spain
Congo	Kenya	Sri Lanka
Costa Rica	Korea, Republic of	Sudan
Cyprus	Lesotho	Sweden
Denmark	Malawi	Switzerland
Dominican Republic	Malaysia	Syria
Ecuador	Malta	Thailand
Egypt, Arab Rep.	Mauritius	Togo
El Salvador	Mexico	Trinidad and Tobago
Finland	Nepal	United Kingdom
France	Netherlands	United States
Gambia, The	New Zealand	Uruguay
Germany	Nicaragua	Venezuela
Ghana	Niger	Zaire
Greece	Norway	Zimbawe

Figure 2:

C

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