Public and Private Heterogenous R&D, and Growth

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Abstract

The empirical analyses show that public and private R&D are strongly intertwined. In particular, the existence of large direct spillovers from public funded R&D to private industry has extensively proven. From an institutional point of view, to stimulate the technology transfer from publicly funded R&D programs to private industry the U.S. adopted an uniform patent policy for public funded research, such as that guaranteed by the Bayh Dole Act. This paper contributes to explain this empirical evidence. Within a neo-Schumpeterian endogenous growth model, it is shown that the intellectual appropriation share of a new commercial valuable idea by private firms and the subsidy of private R&D cost are two equivalent ways to stimulate private R&D effort, and they affect in the same way the endogenous per capita output growth rate. The existence of a trade-off between the per capita output growth rate and level has found. The results show that once IPR are granted to public innovations, a different regime of patent protection should be set for private and public innovations. In particular, patents should only be granted to very innovative and fundamental public ideas.

Keywords: Intellectual Property Rights, Private and Public R&D, Growth

JEL classification: O31, O34, O38

1 Introduction

The recognition of an important role of private and public Research and Development (R&D) effort for the economic performance of a country comes from both academic and non-academic analyses (see the National Science Foundation reports). Moreover, the existence of strong interrelations between public and private R&D has extensively proven by the empirical evidence. Narin *et*

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al. (1997), and McMillan *et al.* (2000) show that, for the U.S. industry, relying in external sources of knowledge centers on public science. In particular, Narin *et al.* (1997) show that during the 1993-1994, 73% of the scientific paper cited by U.S. industrial patents were firm public science sources. David *et al.* (2000) show that public R&D may generate social benefits in the form of knowledge and training spillovers. This proves that private firms benefit of large spillovers from public discoveries and innovations (direct spillovers), whatever is the effect on the private innovative effort. Moreover David *et al.* (2000) find that these spillovers "are held to enhance private sector productive capabilities, and, specifically, to encourage applied R&D investments by firms that lead to technological innovations - from which will flow future streams of producer and consumer surpluses."[1] From a theoretical point of view these spillovers are explained through both the intrinsic nature of knowledge as a nonrival input (see Arrow 1962) and the Merton's issue of priority of scientific discoveries.[2]

Furthermore, since the '80s the U.S. adopted several legislative and institutional arrangements to reinforce the ties between public and private R&D, and to spur the technology transfer of discoveries and inventions from public research programs to private industry. Among the several legislative acts the most influential has been the Bayh Dole Act of the 1980. This Act instituted an uniform federal patent policy for universities and small businesses under which they obtained the rights to any patents resulting from grants and contracts funded by any federal agency and to licence these patents on an exclusive or non-exclusive basis.[3] Based on the belief that legislative arrangements such as the Bayh Dole Act enhance the technology transfer and the academic contributions to innovation and growth in the U.S., similar legislation is being considered in other OECD countries (OECD, 2002).

Therefore, the empirical evidence shows that private and public R&D efforts are strictly intertwined. On the one hand the existence of large direct spillovers from public R&D to private industry has proven. Moreover, the institutional set-up of an economy in the form of intellectual property rights (IPR) can affect the ties between private and public R&D.

[1] The same authors maintain: "Several recent econometric studies, for example, document positive, statistically significant 'spillovers' effects via the stimulation of private R&D investment by publicly funded additions to the stock of scientific knowledge." Yet, a complementarity relationship between public and private R&D investments is not general, in fact the empirical analysis also shows the existence of a substitutability relationship between private and public R&D investment. Spinesi (2007) shows how the mixed empirical evidence on the complemenatarity-substitutability relationship between public and private R&D investment can be explained within a neo-Schumpeterian endogenous growth framework.

[2] Merton (1973) argued that - within the non market rewards structure - the goal of scientists is to establish the priority of discovery by being the first to announce an advance in knowledge. Therefore, the rewards to priority are the recognition awarded by the scientific community for being the first.

[3] Others legislative acts in such direction are the Stevenson-Wydler Technology Innovation Act of the (1980), the Small Business Innovation Development Act (1982), the National Cooperative Research Act (1984), the Federal Technology Transfer Act (1986), the National Cooperative Research and Production Act (1993), the Technology Transfer and Commercialization Act (2000).

This paper considers these interrelations between public and private R&D by considering the existence of IPR for both private and public innovations. The results show that the mere introduction of some form of IPR for public R&D can increase the growth rate of a country. Yet, the strength of the IPR for public R&D has also found to be an important aspect. I argue that the existence of IPR can be beneficial for growth performance but that IPR should be only granted to some fundamental and radical public innovation. In particular, two different regime and rules for patent grants should be introduced for public and private innovations. Some empirical evidence corroborate this policy implication. In fact, although the introduction of some form of intellectual property rights for public R&D can spur the technological transfer from public to private institutions, Cohen *et al.* (2002) find that the most important channels to access publicly funded research are publications, conferences, informal interactions rather than more institutional channels such as patents, licenses, and cooperative ventures. These results refer to all industrial sectors, even the high-tech industries.[4] Therefore, the policy recommendation to only grant patens to radical basic innovations does not limit or impede the technological transfer from universities to industry.

In this paper a neo-Schumpeterian growth framework à la Aghion and Howitt (1992, 1998), and Howitt (1999, 2000) is adopted. Large part of the neo-Schumpeterian description of policy intervention in the R&D process concerns subsidies to private research firms, without any direct intervention of the government in the R&D sector.[5]

This paper distinguishes in the description of the R&D sector, where the basic and the development stage of the research activity are considered.[6] In this framework private firms can conduct both the basic and development stage of the R&D activity. Public research introduces new basic discoveries a share of which can be usefully developed by the private industry to create a new commercial valuable product. This share is assumed to be stochastic and it produces heterogeneous spillovers on the private research firm among the existing product lines. The empirical evidence shows that the development stage is

[4] Cohen *et al.* (2002) find that the pharmaceutical industry more heavily conveys public research knowledge through patents and licenses. However, the same authors maintain that even in this industry informal channels and open science are still more important in conveying public R&D discoveries.

[5] The theoretical conclusions are not univocal. Some theroretical and empirical anlyses conclude that policy has positive effect on both per capita output growth rate and on per capita output level. The alternative view concludes that policy is ineffective on per-capita output growth rate, even if it can produce positive effects on the per capita output level (see Jones 1995, 2005). Although a recent empirical analysis by Ha and Howitt (2006) seems to consider public policy effective even on the per capita growth rate of countries, a conclusive result, both theoretical and empirical, can not be again obtained.

[6] The Science and Engineering Indicators (SEI, 2006) by the National Science Foundation defines as basic the research aimed "to gain more comprehensive knowledge or understanding of the subject under study without specific application in minds". The development stage is defined as "the systematic use of the knowledge or understanding gained from research directed toward the production of useful materials, devices, systems, or methods, including the design and development of prototypes and processes."

the prominent activity of the private research effort.[7] Because of this evidence the drastic, yet realistic, assumption that public R&D only consists in basic research programs is introduced.

Recently, a paper by Cozzi and Galli (2007) focuses on the sequential nature of the innovation process within a dynamic general equilibrium framework. Their paper considers a two stage innovation process, and it evaluates the conditions under which intellectual property rights should be extended to basic discoveries that does not have an immediate marketable application and a commercial value. Cozzi and Galli (2007) show that a pro-growth policy consisting into guarantee an intellectual property protection for 'basic half-ideas' was at the ground of the reforms undertaken in the U.S. around the '80s.

This paper studies the macroeconomic implications of the interplay between private and public investments in R&D, and it complements Cozzi and Galli's (2007) contribution because it also focuses on the strength and width of patents granted to public R&D. The results show that the subsidy to private R&D costs and the intellectual appropriation share of a commercial valuable idea by private R&D firms are two - in some sense - equivalent ways to finance the private R&D effort. In fact, both these ways increase the private innovative effort and the per capita output growth rate, while they reduce the per capita output level. Yet, the subsidy of the private R&D cost concerns the certainty aspect of a R&D process, while the intellectual appropriation's share of a commercial valuable idea concerns the uncertainty aspect of a R&D process. Furthermore, the subsidy and the intellectual appropriation have deep differences from an institutional point of view. The intellectual appropriation of new ideas concerns both public and private research activity, and the policy 'design' of the intellectual property rights involves the political, executive, legal, and jurisprudential powers of a country. The subsidy to private research cost does not deeply shape the institutional set-up of a country as the intellectual property rights policy 'design' does, and it can also be used as a fine-tuning policy instrument. The results are compatible with Cozzi and Galli (2007) findings. In fact whenever any form of intellectual property rights exists for publicly funded R&D programs, the introduction of IPR for public basic ideas spurs the private innovative effort and determines a higher per capita GDP growth rate than what could be obtained from a higher subsidy to private R&D cost. Therefore, the mere introduction of some form of IPR regime for public innovations reveals to be an efficient way to stimulate both the private innovative effort and the per capita output growth rate of a country. However, the second main policy implication is that - once IPR are introduced for public innovations - a different regime for private and public innovations should be introduced, in which patents to public innovations should be only granted to some radical and very innovative ideas.

[7] The SEI states: "The federal government, estimated to have found 61,8% of U.S. basic research in 2004, has storically been the primary source of support for basic research...Industry devoted only an estimated 4,8% of its total R&D support to basic research in that year." (SEI 2006, ch.4 p.13). In addition, "The development of new and improved goods, services, and processes is dominated by industry, which performed 90.2% of all U.S. development in 2004." (SEI 2006, ch.4 p.13).

In fact, the existence of IPR for public innovations generate a further R&D cost for private firms to acquire the licenses from public institutions.

The paper is organized as follows. Section 2 sets up the model, section 3 describes the general equilibrium results, section 4 concludes.

2 The Model

This economy is composed of a final good sector, of an intermediate good sector, and of a R&D sector. As in Aghion and Howitt (1992, 1998), competitive firms produce a homogeneous final consumption good by using a fixed and constant input, and a continuum of intermediate goods with heterogeneous productivity. Intermediate firms produce a continuum of products denoted by N_t at a given time $t \geq 0$. The mass of intermediate goods is continuously enlarging thanks to serendipitous imitation as in Howitt (2000). The manufacturing sector is characterized by free entry and exit, and by a constant returns to scale technology: workers can be hired by a continuum of firms that produce their intermediate goods on a one to one basis from labor. Legally imposed distortions render each of them a local monopoly: this is due to the Patent System. According to the standard Schumpeterian approach à la Aghion and Howitt (1992 and 1998), new intermediate goods are patented and each monopoly is challenged by outsider R&D firms trying to invent and patent a better product and - due to instantaneous price competition - drive the former monopolist out of the market.

The R&D sector is composed of both private and public research activity. Private R&D consists into upgrade the quality (or the production process) of an intermediate product (vertical innovation). As said above, a perfectly enforceable patent law allows the researcher to gain monopolistic rents for all the effective duration of the patent, because - as usual in neo-Schumpeterian growth models with vertical innovation - the incumbent monopolist can be replaced by the next innovator in the same product line.[8] Therefore, it generates the Schumpeterian creative destruction effect. The existence of a perfect stock market channels consumer savings to firms engaged in R&D. Moreover, the government employes skilled workers to obtain basic innovations and discoveries. A stochastic share of the basic ideas can be usefully developed by private R&D firms to find a new commercial valuable innovation and to introduce new intermediate goods. Moreover, the spillovers basic ideas produce on private research firms are heterogeneous between the product lines. According to the legislative acts mentioned in the introduction, basic innovations has granted of intellectual property rights.

[8] See Aghion and Howitt (1992, 1998), Segerstrom (1998), Howitt (1999). Cozzi (2006) proves that the standard neo-Schumpeterian growth models are compatible with a positive and finite R&D investment by the incumbent monopolistic firms. The analysis of this paper is also compatible with Cozzi's (2006) findings.

2.1 Basic Framework

Let us assume continuous time and unbounded horizon. In this economy a mass $L_t > 0$ of infinitely lived families exists. Each family has an identical preference for non-negative consumption flows represented by the intertemporal utility function $\int_0^\infty e^{-rt} C_t dt$, where C_t is the non negative consumption flow of each household. Moreover, each family is endowed with a unit mass of flow labor time bearing no disutility; $r > 0$ is the common and constant subjective rate of time preference. Population growth is constant and equal to $g_L > 0$. The labor market is perfect and the inelastic supply of labor L_t is instantaneously employed by manufacturing firms and by the R&D sector. Capital markets are assumed to be perfect; the linear instantaneous utility implies constant real interest rate always equal to r.

Final output is produced by perfectly competitive firms combining the fixed factor with a large variety of intermediate goods, that is:

$$Y_t = M^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} di \tag{1}$$

with $0 < \alpha < 1$. x_{it} is the amount of intermediate good i produced and used as an input at a given time $t \geq 0$, and A_{it} is the productivity parameter of the current version of that good. M is the constant aggregate mass of fixed factor (such as for example, "land, minerals, oils", etc.). $N_t \in [0, \infty)$ denotes the mass of intermediate goods already invented in the economy at date $t \geq 0$. Since in each sector instantaneous Bertrand competition guarantees that only the most advanced patent holder will be producing, N_t also denotes the mass of active intermediate good industries. The elasticity of substitution between intermediate products is equal to $\varepsilon = \frac{1}{1-\alpha} > 1$.

The perfectly competing R&D firms try to achieve and appropriate the next generation of any intermediate good (vertical innovation process). According to Aghion and Howitt (1998), and Howitt (1999) we consider the leading-edge technology, with an economy-wide leading edge productivity parameter $A_t^{\max}$ that exerts positive R&D spillovers in all intermediate goods. When a new commercial valuable discovery is introduced into an intermediate good sector (a better quality of that intermediate good is introduced) the productivity parameter A_{it} in that sector jumps to $A_t^{\max}$. This specification incarnates Aghion and Howitt's (1998 ch. 3) and Howitt's (1999) inter-sector knowledge spillovers.

The technological frontier $A_t^{\max}$ grows deterministically at a rate proportional to the per product line rate of vertical innovations. The Poisson arrival rate of vertical innovations in any product line i is $\lambda_A l_{Ait} \tilde{f}_i(b_t)$. λ_A is a productivity factor, $l_{Ait} = \frac{L_{At}}{N_t}$ is the per product line research labor time, the function $\tilde{f}_i(b_t)$ captures the effect of the per product line stock of basic knowledge b_t into generate a new patentable and commercial valuable idea in the product line i (see the Appendix A, point 1). The spillovers are heterogenous between the intermediate product lines. As the economy develops an increasing number of intermediate goods, an innovation of a given size in any product line will have a smaller impact on the aggregate economy; hence the marginal impact of each innovation on the stock of public knowledge will be $\frac{\sigma}{N_t}$, where $\sigma > 1$ is a proportionality factor. The aggregate flow of vertical innovations is the number of intermediate goods N_t times the expected flow of vertical innovations per industry line. The economy-wide rate of vertical technological progress is described by the following:

$$g_{At} = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \frac{\sigma}{N_t} \int_0^{N_t} \lambda_A l_{Ait} \tilde{f}_i \left(b_t \right) di \tag{2}$$

Notice that the stock of basic knowledge accumulated over time, and not only the flow of new basic discoveries generates spillovers. Hence, the stock of basic ideas can be used for the development of different versions of an intermediate product.[9] Moreover, the generic specification of $f_i(\cdot)$ leaves room to many ways through which the stock of basic knowledge affects the productivity of private R&D. Therefore, a new better quality version of any intermediate product is the result of private innovation that renders marketable and commercial valuable the offsprings of both public and private research effort.

According to this framework, in equilibrium we will observe an ever-evolving intersectoral distribution of the absolute productivity parameters A_{it}, with values ranging from 0 to $A_t^{\max}$. Defining $a \equiv \frac{A_{it}}{A_t^{\max}}$, we can concentrate on the relative intersectoral distribution, that - as shown in Aghion and Howitt (1998, ch. 3) and in Howitt (1999) - converges to the unique stationary distribution of relative productivity parameters - a - characterized by cumulative distribution function $H(a) = a^{\frac{1}{\sigma}}$, with $0 \leq a \leq 1$. Every time a better quality of an intermediate good is introduced into the economy, the absolute distribution will be re-scaled rightward because the technological process rises to $A_t^{\max}$.

The mass of intermediate products grows as a result of serendipitous imitation, not deliberate innovation.[10] Each person has the same propensity to imitate $\beta > 0$, thus the aggregate flow of new products is:

$$\dot{N}_t = \beta L_t \tag{3}$$

Sine population grows at the constant rate g_L, the number of workers per product line $\frac{L_t}{N_t}$ converges monotonically to $\frac{g_L}{\beta}$.

2.2 Asset Market, Manufacturing, and Vertical R&D

The commercial value of a new intermediate product is given by the firm's expected stock market value that monopolizes the commercialization of the new intermediate product. Let V_t be the expected stock market value of a

[9] Basic ideas in some product lines, such as biotechnology, engineering, electronics, etc., can have an immediate market application, so that the spillovers basic ideas produce for the development of new marketable products in such industry lines can be very high. Instead, basic ideas in ares such as economics, literature, antropology, astronomy, etc., can have a far less useful market application, so that their spillovers can be far more low.

[10] See Howitt (2000). In Howitt (1999), and Cozzi and Spinesi (2005) horizontal innovation is motivated by the same profit seeking objectives as quality improving innovation. The results of this paper are not qualitatively affected by this specification for horizontal innovation.

new intermediate product with maximum productivity $A_t^{\max}$. In this setting the patented basic ideas are developed by private R&D firms to find a new intermediate product. Therefore, each rational private firm pays for the use of patented public basic ideas and therefore it will appropriate a share of the commercial value of the new intermediate product. The share of the market value respectively appropriated by the private R&D firm and by the public research unit that have contributed to introduce the new intermediate product is described as a Nash-bargaining solution between these two forces. Let V_t^p be the expected stock market value of a new intermediate product appropriated by the private R&D firm, and let V_t^b be the expected stock market value of a new intermediate product appropriated by the public research unit, with $V_t = V_t^p + V_t^b$. Therefore, the expected stock market value appropriated by private and public innovators is the solution to the following:

$$\begin{aligned} & \max_{V_t^p, V_t^b} \left(V_t^p\right)^{\phi_i} \left(V_t^b\right)^{1-\phi_i} \\ s.t.\ V_t &= V_t^p + V_t^b \qquad (4) \end{aligned}$$

The solution to this problem gives $V_t^p = \phi_i V_t$, and $V_t^b = (1-\phi_i) V_t$. The parameter $\phi_i \in (0,1)$ represents the institutional set-up in which the bargaining process takes place. ϕ_i indicates that a private R&D firm pays to use an array of basic ideas which are granted of some form of IPR. The existence of laws such as the Bayh Dole Act - and of other legislative arrangements - heavily contribute to determine the value of the parameter ϕ_i in the economy. [11] The parameter ϕ_i is heterogeneous between the product lines. This heterogeneity is explained because of the heterogeneous spillovers between the product lines. In fact, when basic ideas generate a high spillover on the industry line i - i.e. there is a high value for function $\tilde{f}_i(b_t)$ - each firm targeting the industry line i is willing to pay a higher price to develop such patented basic ideas, therefore ϕ_i will be correspondingly lower. This implies that when the same basic idea can be usefully developed by more than one industry line, the government will license that patented idea at the industry line that gain a higher spillover. Moreover, because of the symmetry of the private R&D firms within an industry line, each firm in any industry line i is willing to pay exactly the same price of any other firm in the same industry line.

[11] As shown by Cozzi and Galli (2007) the intellectual property rights for public basic ideas can better off the growth performance of a country. In this setting, the tightness, the width and the ease of the intellectual property rights regime - as measured by ϕ_i - has studied. A low value of ϕ_i can indicate that it is extremely easy for public R&D to obtain patent grants for any basic innovation. This implies that private firms must pay to also use basic discoveries that have a very low innovative power. When $\phi_i \to 1$ basic research programs do not appropriate any share of the expected market value of a new intermediate product. When this happens the Poisson arrival rate of innovation along each intermediate product line can be strictly lower than $\lambda_A l_{At} \tilde{f}(b_t)$. In this case the technological frontier growth rate will be strictly lower than $g_{At} = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma \lambda_A l_{At} \tilde{F}(b_t, \Theta)$. Therefore, in this setting the mere introduction of intellectual property rights for public funded R&D can better off the growth performance.

Applying Aghion and Howitt's (1992 and 1998) methods, the intermediate good i production level that maximizes the monopolist profits at time t is

$$x_{it} = M\left(\frac{\alpha^2 A_{it}}{w_t}\right)^{\frac{1}{1-\alpha}},$$

because the distribution of relative productivities is unchanging, we do not classify the sectors by their index i but by their relative productivity $a \equiv \frac{A_{it}}{A_t^{\max}}$. Defining the productivity-adjusted real wage as $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ and normalizing the fixed factor to one (that is positing $M = 1$) the instantaneous labor demand function for a sector with relative productivity a at date t is rewritten as:

$$\tilde{x}_{it}\left(\frac{\omega_t}{a}\right) = \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

where $\tilde{x}_{it}\left(\frac{\omega_t}{a}\right)$ is a labor demand function for the manufacturing firm. The labor force employed in the manufacturing sector negatively depends on the productivity-adjusted real wage.

The R&D is a perfectly competitive sector, with free entry and exist. Each vertical R&D firm targeting an intermediate product i chooses its R&D intensity to maximize $\phi_i V_t \lambda_A l_{Ajt} \tilde{f}_i(b) - (1-s) w_t l_{Ajt}$, where l_{Ajt} is the labor time flow employed by the vertical R&D firm j at time t, s is the subsidy to private research. Rational individuals and firms know they will appropriate a fraction ϕ_i of the expected stock market value of a patentable and commercial valuable idea in the product line i. The solution to the above problem is $\phi_i V_t \lambda_A \tilde{f}_i(b) = (1-s) w_t$. Notice that, the per product line basic stock knowledge b is taken as given by each individual and firm. Because each R&D firm can invest in any product line, the same first order condition for a maximum profit must hold along any product line $k \neq i$, that is $\phi_k V_t \lambda_A \tilde{f}_k(b) = (1-s) w_t$.

In equilibrium, each R&D firm must be indifferent to invest in any intermediate product line. The industry lines that benefit from a higher spillovers - i.e. have a higher $\tilde{f}_i(b)$ - will pay a higher price to use the patented basic ideas and therefore they will appropriate of a lower share ϕ_i of the expected stock market value V_t. This implies the following no-arbitrage equation between the existing industry lines:

$$\phi_i V_t \lambda_A \tilde{f}_i(b) = \phi_k V_t \lambda_A \tilde{f}_k(b) \tag{6}$$

that implies $\tilde{f}_i(b) = \tilde{f}_k(b)\frac{\phi_i}{\phi_k}$.

We will focus on the symmetric steady state, that is $x_{it} = x_t$, $l_{Ait} = l_{At}$, etc., for every intermediate product line i.[12] In the multisector economy the R&D arbitrage condition is similar to Aghion-Howitt (1998, ch.3, Appendix):

[12] As proven by Cozzi (2005), Howitt's (1999) model admits a continuum of symmetric balanced growth paths.

$$\frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)} w_t = A_t^{\max} \int_0^{\infty} e^{-(r+g_A/\sigma)\tau} \tilde{\pi}\left(\omega e^{g_A \tau}\right) d\tau = $$
$$= A_t^{\max} \int_0^{\infty} e^{-(r+g_A/\sigma)\tau} \frac{1-\alpha}{\alpha} \omega_t \tilde{x}(\omega) e^{-\frac{\alpha}{1-\alpha} g_A \tau} d\tau \qquad (7)$$

On the left hand side of eq. (7) the probability of appropriating the new innovation by the author has been considered. On the right hand side of eq. (7) the discount rate $(r + g_A/\sigma)$, and the profit flows $A_t^{\max} \tilde{\pi}\left(\omega e^{g_A \tau}\right)$ accruing to a successful innovator from date t to infinity have been considered.

2.3 Public R&D

Population differs in the basic research ability, while there are no quality differences among workers employed in vertical R&D and in manufacturing. Let us $G(\theta)$ be the distribution of the 'basic research ability' θ, with θ taking value on $\left[0, \bar{\theta}\right]$ and $\bar{\theta} < +\infty$. The usual properties $G^{'}(\theta) > 0$, $G(0) = 0$, $G\left(\bar{\theta}\right) = 1$ apply. Since each worker must be indifferent between manufacturing and vertical research activity, it will be $w_t = \frac{\phi_i \lambda_A \tilde{f}_i(b)}{(1-s)} V_t$. The additional no-arbitrage condition between improving/manufacturing and basic research effort can be written as:

$$w_t = \frac{\phi_i \lambda_A \tilde{f}_i(b)}{(1-s)} V_t = \lambda_B \varphi \left[E\left(\frac{A_{it}}{A_t^{\max}} \right)^{-1} \right] \theta_0 w_t = \tilde{\lambda}_B \theta_0 w_t \qquad (8)$$

where the function $\varphi\left[E\left(\frac{A_{it}}{A_t^{\max}}\right)^{-1}\right] \neq 0$ represents the spillovers from vertical innovation to basic research, and $\tilde{\lambda}_B \equiv \varphi(1+\sigma)\lambda_B$.[13] The left hand side of eq. (8) indicates the individual expected returns from improving the quality of an intermediate product, which in equilibrium must be equal to the manufacturing wage. The right hand side of the last part of eq. (8) indicates the expected flows return to be employed in basic research programs.

Let us denote θ_0 the threshold value of the 'basic research ability' that satisfies equality (8): θ_0 ability researchers are indifferent between trying to improve the quality of one of the existing intermediate goods, to be employed in basic research, and to be employed in the manufacturing sector. The higher the basic research talent an individual is endowed with, the higher the gain to be employed in basic research programs. The no-arbitrage equation (8) determines the threshold ability value

[13] $\varphi(\cdot)$ can be any positive function of the average relative productivity $E\left[\left(\frac{A_{it}}{A_t^{\max}}\right)\right] = (1+\sigma)^{-1}$. This means that the results of the model hold even if the introduction of a new intermediate product renders the flow of new basic discoveries more complex over time. It is assumed that $\bar{\theta} > \frac{1}{\tilde{\lambda}_B}$.

$$\theta_0 = \frac{1}{\tilde{\lambda}_B}, \tag{9}$$

which is constant along the BGP.[14] Each individual endowed of a research ability $\theta > \theta_0$ will find it profitable to be employed in basic research programs. Hence, in such an economy, for $\theta > \theta_0$, $[1 - G(\theta_0)] L_t$ individuals will choose to be employed in basic research programs. Instead, the individuals endowed with an ability $\theta \leq \theta_0$, that is $G(\theta_0) L_t$, will decide either to introduce a better quality of the existing intermediate goods, or to work in the manufacturing sector. A policy that affects the productivity of basic research effort also affects the threshold ability parameter θ_0. This in turn changes the population employed in basic research programs, and therefore the per product line stock of basic knowledge. This implies that the institutional set-up can affect in different ways the interplay between public and private research effort.

The government conducts basic research programs to accumulate basic knowledge P_t according to the following dynamic law

$$\dot{P}_t = \tilde{\lambda}_B \left[\int_{\theta_0}^{\bar{\theta}} \theta G'(\theta)\, d\theta \right] L_t = \tilde{\lambda}_B m(\theta_0) L_t \tag{10}$$

where $m(\theta_0) L_t \equiv \left[\int_{\theta_0}^{\bar{\theta}} \theta G'(\theta)\, d\theta \right] L_t$ is the average conditioned cumulated basic research effort, $\tilde{\lambda}_B$ is the productivity of each researcher engaged in basic research programs. Eq. (10) implies that the stock of basic knowledge P_t grows at the same rate as the population growth rate g_L.

2.4 Labor and Asset Market Equilibrium

Each researcher endogenously decides to allocate her research labor time to inventive or to manufacturing activity.

Plugging these results in the manufacturing/vertical R&D arbitrage condition (7), and solving the integral yields:

$$\frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)} = \frac{\frac{1-\alpha}{\alpha} \tilde{x}(\omega)}{r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A} \tag{11}$$

Solving the above equation for $\tilde{x}(\omega)$, the labor force employed in the production of the top quality intermediate good is obtained:

$$\tilde{x}(\omega) = \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A \right) \frac{\alpha}{1-\alpha} \tag{12}$$

from which, by inverting eq. (12), it is possible to determine the productivity-adjusted real wage ω_t.

The labor market clearing condition for manufacturing and vertical innovation is:

[14] It assumed that $\frac{1}{\tilde{\lambda}_B} < \bar{\theta} < +\infty$.

$$G(\theta_0) L_t = N_t l_{At} + N_t \int_0^1 \tilde{x}(\omega/a) h(a) da = N_t l_{At} + \frac{N_t \tilde{x}(\omega)}{1 + \frac{\sigma}{1-\alpha}} \tag{13}$$

where $\tilde{x}(\omega/a)$ is the labor demand function of a product line with relative productivity parameter a at the date t, and $h(a)$ is the density function of the cumulative distribution function $H(a)$.

The labor market clearing condition for basic research programs is:

$$[1 - G(\theta_0)] L_t = L_{Bt} \tag{14}$$

which is a constant fraction of the population because the threshold ability parameter θ_0 is constant along the BGP.

From eq. (1), and reclassifying intermediate goods by their relative productivities, the aggregate GDP can be written as (see Aghion and Howitt 1998, ch. 3, and Howitt 1999):

$$\begin{aligned} Y_t &= A_t^{\max} N_t \textstyle\int_0^1 a\tilde{x}(\omega/a)^\alpha h(a) da = \\ &= A_t^{\max} N_t \textstyle\int_0^1 \frac{1}{\sigma} a^{\frac{1}{\sigma}} \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}} da = \frac{N_t A_t^{\max} \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \end{aligned} \tag{15}$$

Notice that, in the light of eq.s (15) and (1), the productivity-adjusted fixed factor rent is:

$$\frac{re}{A_t^{\max}} = (1-\alpha) \frac{Y_t}{M A_t^{\max}} = (1-\alpha) \frac{N_t \left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \tag{16}$$

Therefore, the fixed factor rent increases in the number of intermediate goods, simply because they complement it in the production of the final good; and it decreases in the productivity-adjusted real wage.

3 General Equilibrium

The economy has a unique rational expectation equilibrium on which rational individuals instantaneously jump on. From now onward the time index is eliminated for the sake of notational simplicity.

Let us consider the law of motion of the basic knowledge (10), along the BGP it is obtained:

$$p \equiv \frac{P}{N} = \frac{m(\theta_0) \tilde{\lambda}_B}{\beta} \tag{17}$$

From eq. (2) the productivity growth rate becomes (see the Appendix A, point 2):

$$g_A = \frac{\dot{A}^{\max}}{A^{\max}} = \frac{\sigma}{N} \int_0^{N_t} \lambda_A l_{Ait} \tilde{f}_i(b) di = \sigma \lambda_A l_A \tilde{F}(b, \Theta) \tag{18}$$

Therefore, the labor demand in eq. (12) for the top quality intermediate good becomes:

$$\tilde{x}(\omega) = \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A \right) \frac{\alpha}{1-\alpha} \tag{19}$$

Let us consider both eq.s (19) and (13), along the rational expectation equilibrium, a positive and finite value for the per product line vertical research effort exists:

$$\bar{l}_A = \frac{\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha}}{\left[1 + \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha} \right) \sigma \lambda_A F\left(\tilde{b}, \Theta\right) \right]} \tag{20}$$

From eq. (20) in order to have a positive R&D effort the following condition must hold along the BGP (this condition is similar to condition V of Howitt, 1999 and condition A of Cozzi and Spinesi, 2006)

$$g_L > \frac{\beta}{G(\theta_0)} \frac{r(1-s)}{\phi_i \lambda_A \tilde{f}_i(b) \left(1 + \frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \tag{C}$$

The per capita output is:

$$\frac{Y}{L} = \frac{\frac{N}{L} A^{\max} \left(\frac{\alpha^2}{\omega} \right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} = \frac{\frac{\beta}{g_L} A^{\max} \left(\frac{\alpha^2}{\omega} \right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \tag{21}$$

where eq. (15) has been used. Therefore, the per capita output growth rate is equal to the technological frontier growth rate:

$$g_{Y/L} = g_A = \sigma \lambda_A l_A \tilde{F}(b, \Theta) \tag{22}$$

In the light of eq.s from (17) to (22) the following can be stated:

Proposition 1 *Along the rational expectation BGP, a constant fraction of population is employed in manufacturing, private and public research. Along the BGP, an increase either in the intellectual appropriation parameter ϕ_i proportional in all product lines i or in the subsidy s positively affects the per capita output growth rate and negatively affects the per capita output level.*

Proof. *See Appendix B* ■

The intellectual appropriation of new commercial valuable ideas and the subsidy to private R&D firms are two alternative ways to finance private research effort. Yet, some fundamental differences between these two ways exist. The policy 'design' of the intellectual appropriation of valuable ideas concerns the uncertainty aspect of a R&D process and it involves the political, executive, jurisprudential authorities of a country. Therefore, this policy 'design' strongly shapes the institutional set-up and the environment in which both private and public R&D operate. The subsidy to private R&D does not shape the institutional set-up of the economy as the policy 'design' of the intellectual property rights does, and it only directly affects the private R&D costs and therefore the certainty aspect of a R&D process. Moreover, the subsidy can be also managed in short time horizon.

Given these fundamental differences between the intellectual appropriation parameter and the subsidy to R&D, both these two ways to finance private R&D spur the per product line vertical research effort. However, this effect can be different in magnitude, as the following states:

Proposition 2 *Whenever condition (C1) holds along the BGP - i.e.* $s \geq 1 - \phi_{\min}$ *- a larger subsidy would produce a higher per capita output growth rate and a lower per capita output level than what could be obtained from a higher intellectual appropriation parameter* ϕ_i *in each product line* $i \in [0, N_t]$.

Proof. *See Appendix C* ■

Condition (C1) implies that a marginal increase in the subsidy greatly spurs the private research effort along all the product lines and the per capita output growth rate. Yet, a marginal increase in the subsidy to private R&D firms magnifies the trade off between the per capita output growth rate and the per capita output level. Notice that the higher $\phi_{\min}$ the lower the threshold subsidy s that greatly affects the per capita output growth rate and the per capita output level. This means that the lower is the strength of IPR for public ideas the lower the subsidy to spur the private investment in R&D and the growth rate of the economy.

It seems noteworthy to recall that ϕ_i is a measure of the strength, tightness, width and ease of intellectual property rights granted to public innovations. Therefore, proposition 3 has two main policy implications. When the heterogenous spillovers are strong enough, which is the case from the empirical evidence, a lower IPR strength for public innovations determines a higher effect on the per capita output growth rate than the R&D subsidy. Therefore, once some form of IPR are introduced for public ideas, two different regimes of IPR should be provided for public and private innovations. In particular, patents should be only granted at very innovative and important basic public ideas, while a larger patent protection should be granted to private innovations. In particular, he larger patent protection refers to both patentability requirement and to patent breadth. Therefore, a larger patentability requirement should be set for public basic ideas, while a stronger and larger patent breadth should be set for private innovations[15]

4 Conclusions

Strong intertwined relationships between private and public R&D have documented by the empirical evidence: large spillovers from public research activity

[15] See O'Donoghue, 1998.

towards private industry have widely recognized. Moreover, since the '80s the U.S. adopted several legislative acts to spur the transfer of knowledge and innovations from public funded research programs to private industry. The most influential legislative act has been the Bayh Dole Act of the 1980 that created an uniform federal patent policy that allowed universities and small businesses to retain rights to any patents resulting from government and federal agency funded research and to licence these patents on an exclusive or non-exclusive basis. Based on the belief that legislative arrangements such as the Bayh Dole Act enhance the technology transfer and the academic contributions to innovation and growth in the U.S., similar legislation is being considered in other OECD countries.

This paper investigates on the macroeconomic implications of ties between public and private R&D. To this aim a neo-Schumpeterian growth model à la Aghion and Howitt (1992, 1998) and à la Howitt (1999, 2000) has been adopted. In the R&D sector, public research programs generate basic ideas that do not have an immediate commercial value and application. According to the institutional set-up of the U.S., intellectual property rights are granted to these basic ideas. The private R&D firms appropriate a share of the commercial value of each product, because they pay for the licenses of public basic ideas that can be usefully developed. Moreover, private R&D firms obtain a subsidy for their research costs.

The results of the paper generate two remarkable policy implications. On the one hand this paper shows that the intellectual property rights and the subsidy to private R&D costs are two alternative ways to finance private research effort. Both the intellectual appropriation parameter and the subsidy to R&D spur the private innovative effort. This in turn generates a trade off between the per capita output growth rate and level, by increasing the former and reducing the second. This result seems remarkable because the policy design of the intellectual property rights strongly shapes the institutional set-up of the economy, and it concerns the uncertainty aspect of a R&D process. While the subsidy to private R&D costs does not have an institutional 'weight' as the intellectual property rights does, and it concerns the certainty aspect of a R&D process. The second policy implication suggest the introduction of two different regime of IPR for private and public innovations. In fact, although patents granted to public ideas can spur the technological transfer from universities to industry, they also represent a cost for the private firms. According to these considerations policy should provide IPR to only fundamental and radical public ideas in order to limit the cost incurred by private firm to benefit from the use of basic discoveries.

Finally, this paper shows that any way to finance private R&D - either through subsidy or through lower IPR for public innovations - generate a trade-off between the per capita output growth rate and level.

Appendix A

1. In this part the Poisson arrival rate of vertical innovations has obtained. Let us consider the per product line stock of basic ideas at a given time $t \geq 0$, $p_t \equiv \frac{P_t}{N_t}$. In this framework only a share of basic ideas can be usefully developed by private firms to find new commercial valuable intermediate products. This share is assumed to be a continuous random variable denoted by $b_t \in \left[0, \bar{b}\right]$, with $\bar{b} \leq \frac{m(\theta_0)\tilde{\lambda}_B}{\beta}$, with a cumulative distribution function (cdf) $\Omega(\cdot)$ and density function $\Omega'(\cdot)$. All the product lines i are assumed to have the same cdf and density function for b_t. Private R&D firms gain a positive spillovers from basic ideas that have commercial valuable applications. These spillovers are measured through the function $f_i(\cdot)$ in the product line i. The spillovers functions $f_i(\cdot)$ for each $i \in [0, N_t]$ are assumed bounded above and constants over time. Therefore the expected value of spillovers from commercial valuable basic ideas in the product line i is:

$$\tilde{f}_i(b) \equiv E[f_i(b)] = \int_0^{\bar{b}} f_i(b)\, d\Omega(b) \tag{A1}$$

At a given time $t \geq 0$ in the economy there exist a continuum N_t of product lines. Therefore the per product line Poisson arrival rate of vertical innovation is

$$\begin{aligned} &\int_0^{N_t}\int_0^{\bar{b}} \lambda_A l_{Ait} f_i(b)\, d\Omega(b)\, di = \\ &= \int_0^{N_t} \lambda_A l_{Ait} \int_0^{\bar{b}} f_i(b)\, d\Omega(b) = \int_0^{N_t} \lambda_A l_{Ait} \tilde{f}_i(b)\, di \end{aligned} \tag{A2}$$

2. The no-arbitrage equation in vertical R&D between the product lines imply $\tilde{f}_i(b) = \frac{\phi_k}{\phi_i}\tilde{f}_k(b)$. Therefore the technological frontier growth rate can be rewritten as:

$$\begin{aligned} g_{At} &= \tfrac{\sigma}{N_t}\int_0^{N_t} \lambda_A l_{Ait}\tilde{f}_i(b)\, di = \tfrac{\sigma}{N_t}\lambda_A l_{At}\int_0^{N_t} \tfrac{\phi_k}{\phi_i}\tilde{f}_k(b)\, dk = \\ &= \tfrac{\sigma}{N_t}\lambda_A l_{At}\tilde{F}(b,\Theta)\, N_t = \sigma\lambda_A l_{At}\tilde{F}(b,\Theta) \end{aligned} \tag{A3}$$

where $\tilde{F}(b,\Theta)$ summarizes the sum over the mass of product lines of the spillovers functions, this function is bounded above; $\Theta > 0$ is a proportional factor that also summarizes the shares ϕ_k for each k. Along the BGP the technological frontier growth rate is constant. Q.E.D.

3. By following the same steps as in Aghion and Howitt (1998), the profit flow of any monopolistic firm that manufactures an intermediate product i with productivity A_{it} is

$$\pi_{it} = A_t^{\max}\frac{1-\alpha}{\alpha}\omega_t\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{1}{1-\alpha}} a^{\frac{1}{1-\alpha}} = A_t^{\max}\tilde{\pi}(\omega)\, a^{\frac{1}{1-\alpha}}$$

where $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ is the productivity-adjusted real wage, $\tilde{\pi}(\omega)$ is the profit flow of the intermediate good with the maximum productivity parameter $A_t^{\max}$. The expected stock market value of the last successful R&D firm that has productivity $A_t^{\max}$ is described by the eq. (5) in the text. The expected stock market value of an intermediate product i with absolute productivity A_{it} and relative productivity $\frac{A_{it}}{A_t^{\max}}$ is $V_{it} = V_t a^{\frac{1}{1-\alpha}}$. Therefore, the cumulative expected stock market value of all manufacturing monopolies at a given time $t \geq 0$ is:

$$\int_0^{N_t} V_{it} di = N_t \int_0^1 V_{it} dH(a) = N_t V_t \int_0^1 a^{\frac{1}{1-\alpha}} dH(a) = \frac{N_t A_t^{\max} V_t}{1 + \frac{\sigma}{1-\alpha}} \tag{A3}$$

Appendix B

The first part of this Appendix proves the effect of a higher intellectual appropriation parameter ϕ_i for each product line on the economic performance of a country, $\phi_i \in (0, 1)$ is assumed. The marginal change in the appropriation parameter is assumed to happen proportionally in all the product lines, so that the ratio $\frac{\phi_i}{\phi_k}$ is constant. This proportional change in all the appropriation parameter ϕ_i has interpreted as a lower strength of IPR for public basic ideas. By calculating a marginal change in all parameters ϕ_i, the change has denoted with a generic parameter ϕ. The second part analyses the effect of a higher R&D subsidy.

1. Let us consider the eq. (20). By simply differentiation along the BGP the following is obtained:

$$\frac{\partial l_A}{\partial \phi} = \Delta^{-2} \left\{ \frac{r(1-s)\frac{\alpha}{1-\alpha}\lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}{\left[\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)\right]^2}\Delta - \left[\frac{L}{N} G\left(\theta_0\right) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha}\right] * \left[\frac{(1-s)\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \frac{\partial \tilde{F}(b,\Theta)}{\partial \phi}\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right) - (1-s)\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \tilde{F}(b,\Theta)\lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)}{\left[\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)\right]^2}\right]\right\} \tag{B1}$$

where $\Delta \equiv \left[1 + \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha}\right) \sigma \lambda_A \tilde{F}\left(b, \Theta\right)\right] > 0$, and condition (C) holds. Because the spillovers are assumed constant in this analysis, then $\frac{\partial F\left(\tilde{b},\Theta\right)}{\partial \phi} = \frac{\partial\left[\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k(b)dk\right]}{\partial \phi} = 0$, because we impose a proportional marginal change in each appropriation parameter ϕ_i such that $\frac{\phi_k}{\phi_i}$ remains constant. In fact:

$\frac{\partial \tilde{F}(b,\Theta)}{\partial \phi} = \frac{\partial\left[\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k(b)dk\right]}{\partial \phi} = \frac{\partial\left[\frac{1}{\phi_i} \int_0^{N_t} \phi_k \tilde{f}_k(b)dk\right]}{\partial \phi} = -\frac{1}{\phi_i^2} \int_0^{N_t} \phi_k \tilde{f}_k\left(b\right) dk + \frac{1}{\phi_i} \int_0^{N_t} \tilde{f}_k\left(b\right) dk =$

$= \frac{1}{\phi_i} \left[\int_0^{N_t} \tilde{f}_k\left(b\right) dk - \frac{1}{\phi_i} \int_0^{N_t} \phi_k \tilde{f}_k\left(b\right) dk\right] = \frac{1}{\phi_i} \left[\int_0^{N_t} \tilde{f}_k\left(b\right) dk - \int_0^{N_t} \tilde{f}_k\left(b\right) dk\right] = 0$ Q.E.D.

In eq. (B1) this variation is simply denoted with $\frac{\partial \Theta}{\partial \phi_i}$ because $\Theta > 0$ summarizes all the ratios $\frac{\phi_k}{\phi_i}$in the integral $\int_0^{N_t} \frac{\phi_k}{\phi_i} \tilde{f}_k\left(b\right) dk$. This implies that eq. (B1) is strictly positive, i.e. $\frac{\partial l_A}{\partial \phi} > 0$. Therefore, along the BGP, a higher intellectual appropriation parameter ϕ increases the per product line private R&D effort.

In order to determine the effects of a higher intellectual appropriation parameter ϕ on the market demand for any existing intermediate good, we use the labor market clearing condition:

$$L = G(\theta_0) L + [1 - G(\theta_0)] L = N\bar{l}_A + \frac{N\tilde{x}(\omega)}{1 + \frac{\sigma}{1-\alpha}} + Nl_B \qquad \text{(B2)}$$

where $l_B = \frac{L_B}{N} = [1 - G(\theta_0)] \frac{L}{N}$ denotes the per product line basic research effort. From eq. (8) a constant threshold ability parameter θ_0 is obtained. Therefore - along the new BGP with a higher intellectual appropriation parameter ϕ - the per product line basic research effort $[1 - G(\theta_0)] \frac{L}{N}$ is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (B1) proves that, along the new BGP, the per product line vertical research effort is higher. Therefore, eq. (B2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher appropriation parameter ϕ determines a lower per capita output level. Q.E.D.

The positive effect of a change in the appropriation parameter ϕ on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial \phi} = \sigma\lambda_A \frac{\partial \bar{l}_A}{\partial \phi} \tilde{F}(b, \Theta) > 0 \qquad \text{(B3)}$$

where the inequality follows from eq. (B1). Q.E.D.

2. This part analyses the effect of a change in the subsidy to private research effort s on the economic performance of the economy; $s \in (0, 1)$ is assumed. From eq. (20) the following is obtained:

$$\frac{\partial l_A}{\partial s} = \Delta^{-2} \left\{ \frac{r\frac{\alpha}{1-\alpha}}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \Delta - \left[\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha} \right] * \left[- \frac{\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma\lambda_A \tilde{F}(b,\Theta)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \right] \right\} > 0 \qquad \text{(B4)}$$

Therefore, along the BGP a positive relationship between the subsidy to private research effort s and the per product line private R&D labor time $\bar{l}_A$ is proven. Q.E.D.

In order to determine the effects of a higher subsidy s on the market demand for any existing intermediate good, the labor market clearing condition (B2) is used. As proven above, along the BGP, the per product line basic research effort is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (B4) proves that - along the new BGP with a higher subsidy to private R&D firms - the per product line vertical research effort is higher. Therefore, eq. (B2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher subsidy s determines a lower per capita output level. Q.E.D.

The positive effect of a change in the subsidy to private research effort s on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial s} = \sigma\lambda_A \frac{\partial \bar{l}_A}{\partial \phi} \tilde{F}(b, \Theta) > 0 \qquad \text{(B5)}$$

where the inequality follows from eq. (B4). Q.E.D.

Appendix C

This Appendix compares the effect of a proportional marginal change in the appropriation parameter ϕ_i in all product lines i with the effect of a marginal change in the subsidy s. Along a new BGP with a larger value of either ϕ_i or s determines a higher per product line private innovation effort and a higher per capita output growth rate. In order to compare the magnitude of these effects it suffices to consider the eq.s (B1) and (B4). Whenever the following condition is satisfied $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$, an increase in the subsidy generates the same economic effects as an increase in the appropriation parameter, but the former are higher in magnitude. Let us define $\phi_{\min} \equiv \min \{\phi_i\}_{i=0}^{N}$. Therefore $\phi_{\min}$ is the product line that appropriates the lowest market value among the existing product lines. From eq.s (B1) and (B4), it immediately follows that $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$ if and only if

$$\left[1 - \frac{1-s}{\phi_i}\right] * \left\{ \frac{r \frac{\alpha}{1-\alpha} \Delta}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} + \right.$$
$$\left. + \left[\frac{L}{N} G(\theta_0) - r \frac{(1-s)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \frac{\alpha}{1-\alpha}\right] \frac{\frac{\alpha}{1-\alpha}\left(\frac{1}{\sigma}+\frac{\alpha}{1-\alpha}\right)\sigma \lambda_A \tilde{F}(b,\Theta)}{\phi_i \lambda_A \tilde{f}_i(b)\left(1+\frac{\sigma}{1-\alpha}\right)} \right\} \geq 0$$

that is always true whenever

$$s \geq 1 - \phi_{\min} \tag{C1}$$

Q.E.D.

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