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Optimal Taxation of Housing Income and Economic Growth

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Abstract - The paper analyses the effects on household utility and on economic growth rate of changes in direct tax rates on incomes and on the value of owner housing in a model of dynamic general economic equilibrium. The results confirm similar findings in the literature, although with some qualifications. If government spending consists of public consumption and transfers to households, in general income from business capital should not be taxed, while the imputed rent from owner housing should be taxed at high rates, in relation both to household utility and the economic growth rate. The question of the tax rate on household income is more complex, because this is positively correlated with the rate of growth, but negatively with household utility. Consequently, in the steady state equilibrium, an increase in the tax rate on the income of households is likely to worsen their utility, whereas in the long-run this negative effect may be off-set by a more rapid growth.

Jel Classification: H21, D90

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1. Introduction

The fiscal treatment of housing is a ‘politically delicate’ problem because housing value makes up large part of household wealth, and society regards as ‘of vital importance’the service furnished by the home. In Italy, the value of the housing stock is estimated four times current GDP[1] and approximately account for sixty per cent of net wealth of the households. By 2005, 87 percent of households owned their own homes and many of 13 percent who rent are younger households owners-in-waiting (Guiso and Jappelli, 2000, Cannari and D’Alessio, 2006)

For this reason, in many countries housing enjoys favourable tax treatment since the return to owner housing (i.e. the imputed rent[2] defined as the value of living in own property for a year) is either untaxed or taxed at relatively low rates, while income from business capital is subject to rather high tax rates (Gavhari (1985), Poterba (1992), Atkeson, Chari and Kehoe (1999), Hendershott and White (2000)). On the other hand, the building industry is little exposed to foreign competition and enjoys privileges that cause inefficiencies, which may push up the prices of new houses and the cost of rented accommodation. Because housing service from both owning or renting is likely the principal ‘wage-good’, an increase in its

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[1] See *Bollettino Economico della Banca d’Italia, no.45, novembre 2005, p.30*. By 2004, the percentage composition of net wealth of Italian households was the following:

Value of housing	57.6
Other real assets	10.1
Financial assets (net of debts)	32.3

During 2005, the value of housing stock is arrived at 60% of the net wealth of Italian households .
As regards the US, the value of the housing stock is around twice as high as GDP, while the net wealth of households is 5.6 times GDP (ibid., p. 11). By 2004, 68 percent of households owned their own homes. (Himmelberg, Mayer and Sinai, 2005)

[2] The annual cost of homeownership, also known as “impute rent”, is the sum of many components: the cost of foregone interest that the homeowner could have earned by investing in other assets, the one-year cost of property taxes national or local, the maintenance cost, the expected capital gain (or loss) during the year, and finally the risk premium to compensate homeowners for the higher risk of owning versus renting (Himmelberg, Mayer and Sinai, 2005, pp74-75)

price (or cost) will affect monetary wages; the profits of firms exposed to foreign competition decrease and accumulation of productive capital and economic growth slow down.

The consequences on the welfare of a reform which imposes an equal tax rate on imputed rent and on business capital income has been examined. Such a reform would produce substantial efficiency gains by reducing distortions in the saving allocation (Turnovsky and Okuyama (1994), Skinner (1996), Baxter and Jerman (1999), Coleman (2000), Gervais (2002), Bye and Avitsland (2003).

However, sicne the return from housing service for its owner is a utility flow, and not a spendable monetary income,[3] the optimum tax rate does not necessarily coincide with the tax rates on capital income. There are substantial differences in the taxation of business capital income and housing income. If the taxing of business income becomes too high, firms are forced to work at a loss, to postpone productive investments and even exit the market. The housing property instead furnishes a utility flow whose marginal utility will be the greater, the smaller the stock of housing available. Moreover it gives monetary income, either effectively in the case of renting, or figuratively in the case of owner-occupation.

The most important point, however, is that assessment in terms of social welfare cannot be restricted to a short run and a static context, but must consider the impact that the tax structure will have on the growth rate. There are many studies in the literature which have examined housing taxation within a dynamic model of general economic equilibrium (Greenwood and Hercowitz (1991), Berovec and Fullerton (1992), Jones, Manuelli and Rossi (1997), MacGrattan, Rogerson and Wright (1997), Cremer and Gahvari (1998), Gomme, Kydland and Rupert (2001), Englund (2003)). In particular, Eerola and Maattanen (2005) analyse a model of general economic equilibrium *à la* Ramsey in which the government finances its expenditure by levying taxes on consumption, work income, business capital income, and imputed rent from owner housing. The optimal tax structure should provide for nil (or even negative) tax rate on business capital income. Given that housing furnishes a service which directly enters the utility function, the tax rate on imputed rent should exceed

[3] In other words, there is no immediate substitutability between the value of the imputed rent from owning a house and the monetary income which can be used to purchase other utility-furnishing goods and services.

that on business capital income. "*…in general the optimal tax rate on the imputed rent should not equal the tax rate on the business capital income. ... both housing and other consumption should be taxed at relatively high tax rates, whereas the tax rate on business capital income should be close to zero*" (Eerola and Maattanen, 2005, p.27).

In my opinion, even though interesting, in this analysis there is a main problem as it considers an exogenous per capita income growth rate equal to the growth rate of technical progress, as is usual in neoclassical models *à la* Ramsey. Moreover, this growth rate is not influenced by public policies, which only affect the allocation of income between consumption and investment in steady-state equilibrium. During transition to the steady state growth can be accelerated or slowed down; in this context only the tax structure is important. However, if we consider infinity living households, the most important factor in their welfare (measured by the current value of future utility flows) is indubitably the income growth rate.

In my opinion the analysis should be conducted in a dynamic model where the steady-state growth rate is endogenous depending on the government's policies. Moreover, the structure of the direct tax rates and public expenditure must be modelled on the basis of the overall tax system of a specific country.

In Italy, housing is subject to two types of tax. As annual income flow, imputed on the basis of the cadastral value of the housing property, it is added to income from work and/or from business capital, and taxed at specific rates (IRE on households income, once a deduction has been made for first-home ownership; IRES on business capital income). As capital value it is subject to a tax on the cadastral value of the housing property (ICI) used to finance local governments spending. The average rate applied to owner-occupied housing therefore derives from the sum of ICI plus the IRE levy on households income.

The article is organized as follows. Section 2 proposes an endogenous growth model with a representative infinity living household that gains utility from private consumption and from the housing services. The model is similar to that proposed by Eerola and Maattanen (2005) but differs from it in that the growth rate depends on the government's fiscal policies. Firms produce private goods and services using a production function à la Rebelo (1991), maximizing short run profits and investing their net income in productive capital. The government levies taxes on the incomes of firms and households, and on rent from housing.

Taxes on consumptions are not considered. The government budget is balanced, so that the tax yield finances both public consumption and transfers to households. Section 3 studies the effects of changes in the direct tax rates on the rate of growth and on the utility of households, starting from an optimal benchmark situation represented by a competitive economy without a government. Section 4 draws the main conclusions on the effects of housing taxation.

2. An endogenous growth model with taxation of rent from housing

2.1 *The model*

Consider the following Cobb-Douglas production function:

$$y = k^{\alpha} h_I^{1-\alpha}$$

where y is per capita net output, k is per capita productive capital stock and $h_I = \gamma h$ is per capita housing stock used as input in the productive process. Employment is normalized to one.

Let the stock of productive capital k(t) be owned by firms and yield an annual return r. Let the housing stock h(t) be owned by households; the share $h_I = \gamma h$ is rented to firms, which use it as productive input. The annual return on housing stock is hypothesized equal to r, i.e. equal to the return on business capital. The return from owing a house comes from the rent the owners saves by living in the house rent-free (dividend) and from house price appreciation over time (capital gain). In long-run competitive equilibrium every asset yields a return (capital gain plus dividend) equal to that on alternative assets. Our hypothesis is that the rate of return of productive capital and housing stock is r exogenous and equal to that prevalent on international real assets market.

Given the unit price of output, firms maximize annual profits with respect to k and h_I:

$$\Pi = k^{\alpha} h_I^{1-\alpha} - rk - rh_I - W$$

where W is the wage rate or unit labour income.

From first order conditions, easily is obtained the following optimal ratio between housing input and productive capital stock:

$$h_I = \frac{1-\alpha}{\alpha} k$$

By substituting the optimal h_I into the production function, the following production function à la Rebelo (1991) is obtained[4]:

(a1) $$y(t) = ak(t) \qquad \text{where: } a = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}$$

The labour income W is calculated in residual manner from the total income once the return on both business capital and housing have been paid.

Households earn the labour income plus the rent obtained from firms leasing a share of the housing stock. The net income of households, once they have paid taxes and received their government transfers, is used for private consumption and to invest in housing because households derive utility both from consumption c(t) and from the housing services $(1-\gamma)h(t)$.

The owners of the firms (or *capitalists*) earn the return of productive capital stock minus the rent paid to family for leasing the housing stock $h_I(t)$. The net income of the owners of the firms, once they have paid direct taxes, is used totally to investments in physical capital; the hypothesis is that the owners of the firms derive no utility from private consumption, but only from capital accumulation. Therefore, capital stock increases over time as a consequence of firms investment[5].

[4] It is assumed that the housing stock is not binding at time t=0 and therefore it is not binding in any future time.

[5] A different hypothesis is assuming that the whole income, net of direct taxation and transfers, accrues to households who decide how to allocate it in consumption, investment in physical capital and investment in housing. However, when capital market imperfections exist and Modigliani-Miller theorem fails, it is advantageous for households to leave all net profits to firms for investments in physical capital saving the cost of financial intermediation.

The national government levies taxes with different proportional average rates on the incomes of firms (IRES) and households (IRE).[6] A share β of public revenues is given back to households as transfers; the remaining share (1- β) is allocated to national public consumption considered unproductive and 'useless', i.e. not included in the households' utility function. It is well known that public spending on productive investments may positively affect economic growth rate, as shown in Barro's (1990).[7] However, I think it right to separate the effects on the growth rate produced by direct taxation from those exerted by public spending, whether productive or unproductive.

Local governments levy taxes with proportional rates on the cadastral value of housing (ICI) and use the revenues to finance local public consumption.

The economic variables not yet introduced have the following meaning:

τ_i proportional average rate of the i-th tax, i= I, F, h

γ share of housing used by firms as productive input

β share of public revenues used for transfers to households

E_i revenues of the national , i=G, and local, i=L, governments

TR_F government transfers to households

CP_i national, i=G, and local, i=L, public consumption

Y_i income of households, i=F, and firms, i=I

YN_i net income of households, i=F, and firms, i=I

[6] Governments obviously also levy indirect taxes on value added and consumption. However, these are not explicitly considered here because they do not directly affect housing income.

[7] For more detailed discussion of the effects of public expenditure on the growth rate see Balducci (2005).

The following national accounting identities and behavioural assumptions hold:

$$y = Y_F + Y_I$$

$$Y_I = rk(t) - \gamma rh(t)$$

$$Y_F = (a\text{-}r)k(t) + \gamma rh(t)$$

$$E_L = CP_L = \tau_h\, h(t)$$

$$E_G = \tau_F\, Y_F + \tau_I\, Y_I$$

$$TR_F = \beta\, E_G$$

$$YN_I = (1\text{-}\tau_I)Y_I = b_{11}k(t) + b_{12}\, h(t)$$

$$YN_F = (1\text{-}\tau_F)Y_F + TR_F - E_L = b_{21}k(t) + b_{22}h(t)$$

where the structural parameters have the following signs

$$\begin{aligned} b_{11} &= (1-\tau_I)r > 0 \\ b_{12} &= -(1-\tau_I)\gamma r < 0 \\ b_{21} &= (1-\tau_F+\beta\tau_F)(a-r)+\beta r\tau_I > 0 \\ b_{22} &= (1-\tau_F+\beta\tau_F-\beta\tau_I)\gamma r-\tau_h \gtreqless 0 \end{aligned}$$

Table 1 sets out the effects of changes of the tax rates on the structural parameters:

Table 1: *Effects on the parameters b_{ij} of changes in the tax rates*

	δb_{11}	δb_{12}	δb_{21}	δb_{22}
$\delta\tau_h$	0	0	0	$-1<0$
$\delta\tau_F$	0	0	$-(1-\beta)(a-r)<0$	$-(1-\beta)\gamma r<0$
$\Delta\tau_I$	$-r<0$	$\gamma r>0$	$\beta r>0$	$-\beta\gamma r<0$

Before proceeding, the assumptions underlying investments must be explained. It is assumed that owners of firms invest all their net income in productive capital. Given that the gross rate of return on the two assets k and h is the same, to be explained is why firms invest in productive capital rather than in housing. This decision depends on the expected rate of return net of tax. Therefore, it is profitable for firms to invest in productive capital if:

$$\frac{(1-\tau_I)(rk - rh_I)}{k} > \frac{(1-\tau_I)rh - \tau_h h}{h} \qquad ,$$

that is if: $\tau_h > (1-\tau_i)r\frac{1-\alpha}{\alpha}$; in words, if the local tax levied on housing is greater than the net return of housing input..

A different explanation is required of why households invest in housing, rather than in productive capital. It is evident that the profitability of one or the other form of investment should be established on the basis of the expected return net of tax. Because owner-occupation directly provides utility as a housing service, it is profitable for households to invest in housing if :

$$\frac{(1-\tau_F)rh - \tau_h h + U(c,h)}{h} > \frac{(1-\tau_F)(rk - rh_I)}{k}$$

which is satisfied if : $\frac{U(c,(1-\gamma)h)}{h} > \tau_h - (1-\tau_F)r\frac{1-\alpha}{\alpha}$.[8]

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[8] It is assumed that this condition holds at time t=0 and is maintained in the steady state equilibrium.

2.2. *The intertemporal optimization problem*

It is assumed that households take as given the policy variables established by the national and local governments, and that they choose the consumption c(t) so to maximize the current value of the utility flows. The utility function is hypothesized of CRRA type, its arguments being private consumption and the housing services:

$$u[c(t),h(t)] = \frac{\left[c(t)^{\sigma}((1-\gamma)h(t))^{1-\sigma}\right]^{1-\vartheta}}{1-\vartheta}$$

The dynamic optimization problem can be written as follows:

$$\max_{c(t)} \int_0^{\infty} e^{-\rho t} u[c(t),h(t)]dt$$

subject to

(1) $\dot{k}(t) = b_{11}k(t) + b_{12}h(t)$ $\qquad k(0)=k_0>0$

(2) $\dot{h}(t) = b_{21}k(t) + b_{22}h(t) - c(t)$ $\qquad h(0)=h_0>0$

and two trasversality conditions: : $\lim_{t\to\infty} \lambda(t)k(t) = 0$ and $\lim_{t\to\infty} \mu(t)h(t) = 0$

2.3. *The optimal static equilibrium.*

Let us suppose that economy is in static equilibrium: no investments are made in either productive capital or housing. Given the budget constraint: : $c = b_{21}k_0 + b_{22}h$, obtained from equation (2), households maximize their utility function U(c,h) by choosing the optimal values of consumption and stock of housing :

$$c^* = \sigma b_{21} k_0$$

$$h^* = -(1-\sigma) b_{22} b_{21} k_0$$

Given that $b_{21}>0$, housing stock h* is positive only if b_{22} is negative. This parameter has two components: the first is income net of taxes obtained by renting a share of their building stock to firms; the second is tax on figurative income from housing services, i.e. the ICI on imputed rent from housing. In the following analysis, we assume that this second effect prevails on the first one and, therefore, that $b_{22}<0$.

Deriving the utility function U(c*,h*) with respect to the *i-it* tax rate, we obtain the following condition:

$$\frac{\partial U(c^*,h^*)}{\partial \tau_i} = (1-\vartheta) U(c^*,h^*) \left[\frac{1-\sigma}{b_{22}} \frac{\partial b_{22}}{\partial \tau_i} + \frac{1}{b_{21}} \frac{\partial b_{21}}{\partial \tau_i} \right]$$

Given the signs of the derivatives described in Table 1 and assumed $\theta<1$, we can straightforwardly calculate the effects of changes in the i-th tax rates on the utility of households. These effects are shown in Table 2.

Table 2: *Short run effects on the households utility of changes in tax rates*

$sign \frac{\partial U(c^*,h^*)}{\partial \tau_h} = -sign \frac{1-\sigma}{b_{22}} > 0$
$sign \frac{\partial U(c^*,h^*)}{\partial \tau_F} = -sign \left[\gamma \frac{1-\sigma}{b_{22}} + \frac{\sigma(a/r-1)}{b_{21}} \right] \leq 0$
$sign \frac{\partial U(c^*,h^*)}{\partial \tau_I} = -sign \left[\gamma \frac{1-\sigma}{b_{22}} - \frac{1}{b_{21}} \right] > 0$

The main conclusions that can be drawn from this static context are the following. The effect on household utility of an increase in the tax on imputed rent from housing is positive. An increase in τ_h reduces the net spendable income of households, so that one would expect utility to diminish (*income effect*); but it advantages private consumption (*substitution effect*) and this substitution effect in general prevails.

Also the effect on household utility of an increase in the tax rate on firm's income is positive and it mainly depends on the national government's distributive policy, i.e. on the share β of public budget used for transfers; it is higher, the greater is β.

The absolute value of the effect of a change in the tax rate on household incomes depends on parameter β also: the higher is β, the lower is the effect on household utility. However, in this case the sign is uncertain. The substitution effect is proportional to the share γ of the housing stock rented to firms as productive input , while the income effect depends on direct reduction of household spendable income. For reasonable values of the structural parameters, it is likely that the negative sign will prevail: that is, in the short run household utility decreases with an increase in direct tax rate.

2.4. *The steady state optimal growth equilibrium*

The problem of dynamic optimization of a representative household requires the maximisation of the following Hamiltonian function with respect to control variable c(t) and state variables k(t) and h(t):

$$H\left[c(t), h(t), k(t), t\right] = e^{-\rho t} u\left[c(t), h(t)\right] + \lambda(t)\left[b_{11} k(t) + b_{12} h(t)\right] + \mu(t)\left[b_{21} k(t) + b_{22} h(t) - c(t)\right]$$

where $\rho > 0$ represents the intertemporal discount rate and λ(t) and μ(t) are the co-state variables. It will choose the optimal consumption path *c(t)* in respect of the dynamic constraints (1) and (2), the non-negativity conditions: k(t)≥0, h(t)≥0, c(t)≥0 and the transversality conditions. We also assume that households consider the tax rate τ_i to be given and constant; in other words, *fiscal policy is exogenous*.

In appendix A this optimization problem is solved and the existence of the following steady state equilibrium growth rate g* is proved:

$$(3) \qquad g^* = b_{11} + \frac{1}{2}\left\{\left[b_{22} - b_{11} - (\frac{c}{h})^*\right] + \left[\left(b_{22} - b_{11} - (\frac{c}{h})^*\right)^2 + 4b_{12}b_{21}\right]^{1/2}\right\}$$

where:

$$(4) \qquad \left(\frac{c}{h}\right)^* = \frac{\sigma}{1-\sigma}\frac{(\rho - b_{11})(\rho - b_{22}) - b_{12}b_{21}}{(\rho - b_{11})} > 0 \quad \text{if}: \rho > b_{11} \text{ (sufficient condition)}$$

2.5. *Endogenous growth rate in absence of governments.*

It is assumed that both national and local governments behave as ‘benevolent planners’. They ensure public balanced budgets and choose the optimal tax rates, deriving them from the Hamiltonian function as first-order conditions:

$$(5) \qquad \frac{\partial H(.)}{\partial \tau_i} = [\lambda(t) - \mu(t)]\left[k(t)\left(\frac{\partial b_{11}}{\partial \tau_i} + \frac{\partial b_{21}}{\partial \tau_i}\right) + h(t)\left(\frac{\partial b_{12}}{\partial \tau_i} + \frac{\partial b_{22}}{\partial \tau_i}\right)\right] = 0$$

In general, the difference between the two co-state variables: $\lambda(t) - \mu(t)$ is different from zero[9]; therefore the first-order condition (5) must be verified by the expression in the square brackets. Given the values of the derivatives set out in Table 1, the following first-order conditions are obtained:

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[9] In fact the two co-state variable vary at the same rate, but their initial values $\lambda(0)$ and $\mu(0)$ are normally different and therefore: $\lambda(t) \neq \mu(t)$, for each t.

(5a) $$\frac{\partial H(.)}{\partial \tau_h} = -h(t) < 0$$

(5b) $$\frac{\partial H(.)}{\partial \tau_F} = -(1-\beta)\left[k(t)(a-r) + \gamma r h(t)\right] < 0$$

(5c) $$\frac{\partial H(.)}{\partial \tau_I} = -(1-\beta)r\left[k(t) - \gamma h(t)\right] < 0 \qquad \text{se : } k_0 > \gamma h_0$$ [10]

The following proposition summarizes the meaning of these conditions:

Proposition 1.
The first-best condition in relation to households utility and rate of growth is in the absence of national and local governments, i.e. for $\tau_h = \tau_F = \tau_I = 0$ [11]

In the absence of governments, and for γ=0, the basic features of the economy are the following:

$$\left(\frac{c}{h}\right)^* = \frac{\sigma}{1-\sigma}\rho \ , \qquad\qquad g^* = r$$

The reason for this result is not difficult to understand. Because tax revenues (those of both national and local governments) are used to finance 'useless' public consumption – i.e. which does not directly create utility – it is obvious that every reduction in disposable income reduces the household utility and investments of the firms.

[10] With reference to Italian economy, the value of housing stock is approximately 4/3 than the value of business capital stock. Therefore, if γ<3/4 , the sign of this inequality is verified.

[11] Because $\frac{\partial H(.)}{\partial \beta} = k(t)(a-r)\tau_F + h(t)\gamma r \tau_I \geq 0$ β is indeterminate if τ_i=0; if one of the tax rates on incomes were positive, then: $\beta=1$.

However, governments are constitutionally obliged to furnish public services like defence, justice, security, health, etc.. Therefore, the result obtained in absence of government is not realistic and could be an ‘ideal’ benchmark at most.

3. Changes in direct tax rates, economic growth rate and households utility

3.1. From equation (4) and using the values set out in Table 1, we can study the effects of changes in direct tax rates on the optimal ratio between consumption and housing stock (c/h*). The signs of these changes are reported in Table 3:

Table 3. *Effects of changes in tax rates on the optimal ratio between consumption and housing stock*

$\frac{\partial (c/h)^*}{\partial \tau_h} = \frac{\sigma}{1-\sigma} > 0$
$\frac{\partial (c/h)^*}{\partial \tau_F} = \frac{\sigma}{1-\sigma} \frac{\gamma r(1-\beta)}{\rho - b_{11}} \left[\rho - a(1-\tau_I)\right] < 0 \Rightarrow \tau_I < 1 - \frac{\rho}{a}$
$\frac{\partial (c/h)^*}{\partial \tau_I} = \frac{\sigma r}{1-\sigma} \left[\beta\gamma - \frac{\gamma b_{21} + \beta b_{12}}{\rho - b_{11}} - \frac{b_{12} b_{21}}{(\rho - b_{11})^2}\right] < 0$

A rise in the tax rate on housing rent increases optimal consumption relatively to housing stock, changing its relative profitability; on the contrary a rise in the tax rates on the income of firms and on the income of households reduces optimal consumption with respect to the housing stock. The only condition is that the capital stock productivity, net of taxes *a(1-*τ_I*),* will be greater than intertemporal discount rate ρ.

3.2. With reference to the tax rate τ_i , i=h,F,I, the effects on the steady state growth rate g* are described by the following very complex condition

$$\frac{\partial g^*}{\partial \tau_i} = \frac{\partial b_{11}}{\partial \tau_i} + \frac{1}{2}\frac{\partial Z}{\partial \tau_i}\left[1 + \frac{Z}{(Z^2 + 4b_{12}b_{21})^{1/2}}\right] + \frac{b_{21}\frac{\partial b_{12}}{\partial \tau_i} + b_{12}\frac{\partial b_{21}}{\partial \tau_i}}{(Z^2 + 4b_{12}b_{21})^{1/2}}$$

where: $Z = b_{22} - b_{11} - (\frac{c}{h})^* < 0$[12]

The effects produced by changes in i-th tax rate are shown in the following Table 4, summarized in Proposition 2 and represented by Figure 1:

Table 4: *Effects of changes in direct tax rates on the economic growth rate g**

$\frac{\partial g^*}{\partial \tau_h} = -\frac{1}{2(1-\sigma)}\left[1 + \frac{Z}{(Z^2 + 4b_{12}b_{21})^{1/2}}\right] > 0$ because: $Z<0$, $b_{12}<0$
$\frac{\partial g^*}{\partial \tau_F} = (1-\beta)\gamma r[\ \frac{1}{2}\left[1 + \frac{Z}{(Z^2 + 4b_{12}b_{21})^{1/2}}\right]\left(-1 + \sigma\frac{\rho - a(1-\tau_I)}{(\rho - b_{11})(1-\sigma)}\right) + \frac{(1-\tau_I)(a-r)}{(Z^2 + 4b_{12}b_{21})^{1/2}}\ ] >< 0$
$\frac{\partial g^*}{\partial \tau_I} = r[-1 + \frac{1}{2(1-\sigma)}\left[1 + \frac{Z}{(Z^2 + 4b_{12}b_{21})^{1/2}}\right]\left(1 - \sigma - \beta\gamma + \sigma\frac{b_{21}\gamma + b_{12}\beta}{\rho - b_{11}} + \frac{b_{12}b_{21}}{(\rho - b_{11})^2}\right) + \frac{b_{21}\gamma + b_{12}\beta}{(Z^2 + 4b_{12}b_{21})^{1/2}}\ ] < 0$

Proposition 2:

A higher tax rate on imputed rent from housing and a lower tax rate on business capital income produce a higher rate of growth; the effect of a change in the tax rate on household income on growth rate is uncertain.

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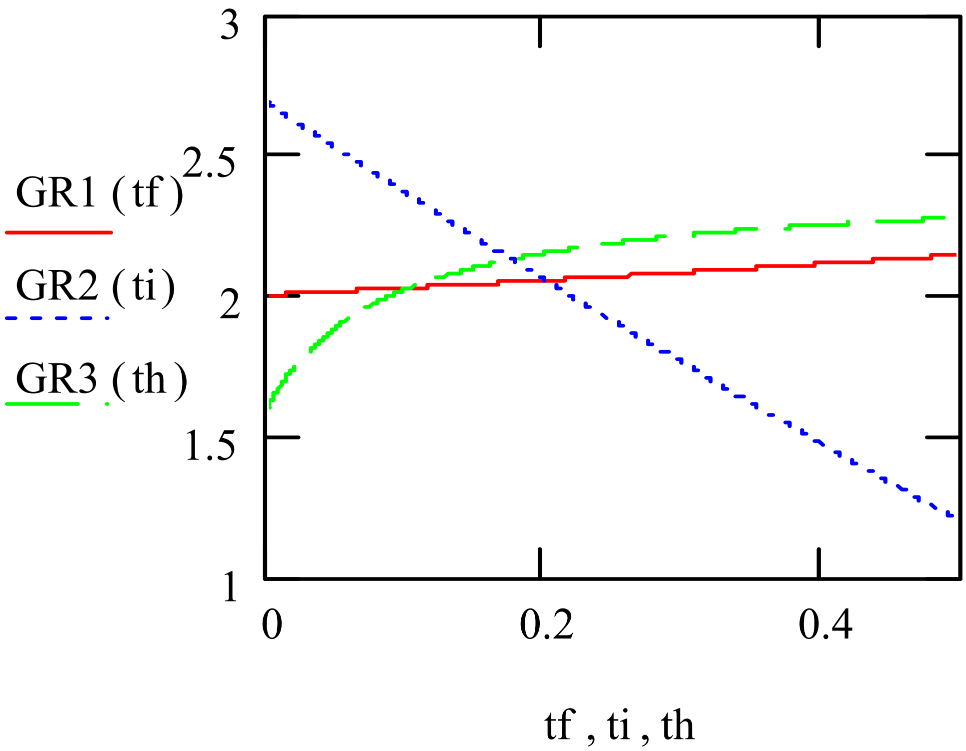
[12] In fact, b_{22} is negative, while b_{11} and (c/h) are positive.

Figure 1: *The growth rate g* in relation to i-th tax rate.*

Parameter values: a=0.30, r=0.03, ρ= 0.04, σ=0.60, γ=0.25, β=0.60



3.3. The utility of a representative household, taking account of the optimal ratio between consumption and the housing stock (c/h)*, is the following:

$$u[c(t),h(t)]=\frac{\left[((c/h)^{*})^{\sigma}(1-\gamma)^{1-\sigma}h(0)e^{g^{*}t}\right]^{1-\vartheta}}{1-\vartheta}$$

The effect of a change in the tax rate τ_i , i=h,F,I, is given by the sum of two components: the change in the optimal ratio between consumption and housing stock (c/h)*, and the change in the housing stock, that grows in the time at the rate g*. In formal terms, it is described by the following condition:

$$\frac{\partial u(.)}{\partial \tau_i} = (1-\vartheta)u(c^*, h^*)\left[((c/h)^*)^{-1} \frac{\partial (c/h)^*}{\partial \tau_i} + \frac{\partial g^*}{\partial \tau_i} \right] \qquad \theta<1$$

Table 5: *Effects of changes in tax rate i-th on household utility*

$\frac{\partial U(c^*,h^*)}{\partial \tau_h}$	*An increase in the tax rate* τ_h *produces both a higher steady-state ratio (c/h)* and a higher growth rate g*; the overall effect on household utility is* ***positive***.
$\frac{\partial U(c^*,h^*)}{\partial \tau_F}$	*An increase in the tax rate* τ_F *reduces the steady-state ratio (c/h)*,while the effect on the growth rate g* is uncertain; the overall effect on household utility is* ***uncertain***.
$\frac{\partial U(c^*,h^*)}{\partial \tau_I}$	*An increase in the tax rate* τ_I *produces a lower steady-state ratio (c/h)* and a lower growth rate g*; the overall effect on household utility is* ***negative***.

Proposition 3:
A higher tax rate on imputed rent from housing and a lower tax rate on business capital income improve household utility; the effect of an increase in the tax rate on household income is uncertain.

Comment is only necessary in regard to the effect on utility of a change in the tax rate on household income. In the static equilibrium, a rise in this tax rate reduces utility by making the negative income effect prevail. However, shifting expenditure by households from private consumption to housing increases the growth rate. Consequently, in the long run equilibrium, the positive effect on growth rate may off-set the initial reduction of the utility, as shown by Figure 2.

Figure 2: *Household utility at time t=100 in relation to tax rate i.*
Parameter values: a=0.30, r=0.03, ρ= 0.04, σ=0.60, γ=0.25, β=0.60.

Figure 2

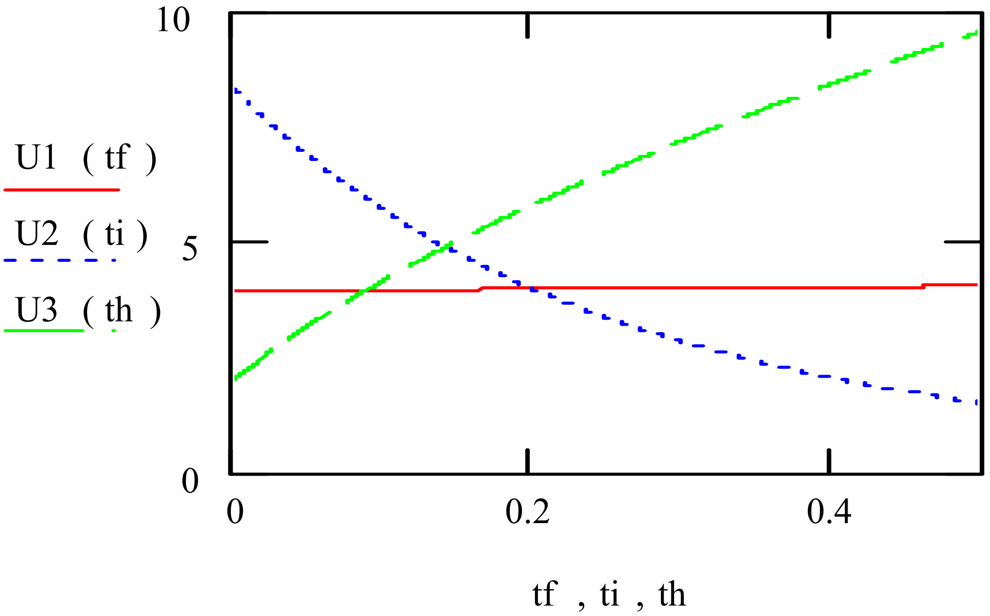


Table 6: *Effects of changes in policy variables on the optimal ratio between the housing stock and productive capital, on the household utility, and on the growth rate.*
Values of parameters: a=0.30, r=0.03, ρ=0.04, σ=0.60

Steady-state values	**(h/k)***	**U*(t=100)**	**g***
β=0.00; γ=0.25; τ_F=0.00; τ_I=0.00; τ_h=0.00	**0.73**	**6.21**	**2.45**
β=0.60; γ=0.25; τ_F=0.00; τ_I=0.00; **τ_h=0.10**	0.43	9.60	2.67
β=0.60; γ=0.25; τ_F=0.00; τ_I=0.00; **τ_h=0.20**	0.31	12.37	2.77
β=0.60; γ=0.25; **τ_F=0.10**; τ_I=0.00; τ_h=0.00	0.72	5.96	2.46
β=0.60; γ=0.25; **τ_F=0.20**; τ_I=0.00; τ_h=0.00	0.70	5.70	2.48
β=0.60; γ=0.25; τ_F=0.00; **τ_I=0.10**; τ_h=0.00	1.00	3.47	2.02
β=0.60; γ=0.25; τ_F=0.00; **τ_I=0.20**; τ_h=0.00	1.32	3.47	1.61
β=0.60; γ=0.25; **τ_F=0.20; τ_I=0.20; τ_h=0.10**	0.55	4.17	2.07
β=0.60; γ=0.25; **τ_F=0.30**; **τ_I=0.30**; **τ_h=0.20**	0.36	4.19	1.91
β=1.00; γ=0.25; τ_F=0.20; τ_I=0.20; τ_h=0.10	0.59	4.17	2.05
β=0.60; **γ=0.25**; τ_F=0.20; τ_I=0.20; τ_h=0.10	0.55	4.17	2.05
β=0.60; **γ=0.50**; τ_F=0.20; τ_I=0.20; τ_h=0.10	0.43	4.20	1.88

4. Conclusion

The paper has examined the effects of changes in the rates of direct taxes on household incomes, firm' incomes , and the value of the housing stock, both in a context of static equilibrium and in an endogenous growth model which takes account of the structure of taxation and public expenditure. The main results obtained in relation to the households utility and the rate of growth confirm similar findings in the literature, with some qualifications.

If public expenditure consists of useless public consumption and of transfers to households, the optimal condition for the economy is the absence of government, i.e. the absence of both national and local taxes and public spending.

Setting aside this unrealistic situation, the main finding of the paper is as follows: the business capital income should be entirely untaxed, while the imputed income from housing should be taxed at high rates, with reference both to household utility and to the economic growth rate. The question of the tax rate on household income is more complex, because this is positively correlated with the growth rate, but negatively with household utility. In this case, a trade-off between the two most important policy objectives of growth and welfare exists, and a compromise solution is necessary.

Appendix A : *Proof of existence of the steady state equilibrium growth rate g**

The analytical solution of the intertemporal optimization problem is obtained by deriving from the Hamiltonian function the following first-order maximum conditions:

(1A) $$e^{-\rho t}u_c = \mu(t)$$

(2A) $$-\dot{\lambda}(t) = b_{11}\lambda(t) + b_{21}\mu(t)$$

(3A) $$-\dot{\mu}(t) = b_{12}\lambda(t) + b_{22}\mu(t) + e^{-\rho t}u_h$$

It is also necessary to impose the following transversality conditions:

$$\lim_{t\to\infty}\lambda(t)k(t) = 0 \quad , \qquad \lim_{t\to\infty}\mu(t)h(t) = 0$$

Using (1A), equations (2A) and (3A) can be rewritten in terms of growth rates of the co-state variables, as follows::

(4A) $$-g(\lambda) = b_{11} + b_{21}\frac{\mu(t)}{\lambda(t)}$$

(5A) $$-g(\mu) = b_{22} + b_{12}\frac{\lambda(t)}{\mu(t)} + \frac{(1-\sigma)}{\sigma}\frac{c(t)}{h(t)}$$

where $g(x) = \dfrac{\dot{x}(t)}{x(t)}$

Because in steady state the growth rates of the variables must be constant, differentiating (4A) with respect to time demonstrates that the following relation must hold:

(6A) $$g(\lambda) = g(\mu)$$

so that also the ratios between the two co-state variables are constant in time. Moreover, in consideration of (6A), conditions (4A) and (5A) can be equalized to obtain:

(7A) $$\frac{c(t)}{h(t)} = \frac{\sigma}{1-\sigma}\left[b_{11} - b_{22} + b_{21}\frac{\mu}{\lambda} + b_{12}\frac{\lambda}{\mu}\right]$$

Differentiating (7A) with respect to time demonstrates that: $g(c) = g(h)$. Likewise, rewriting equation (1) in the text in terms of rate of variation and differentiating with respect to time, one demonstrates that $g(h) = g(k)$ must hold. Considering the previous relations jointly, and recalling the production function: y(t)=a k(t), entails the following relation among the constant growth rates of the economic variables:

(8A) $g(c)=g(h)=g(k)=g(y)=g$

Rewriting the maximum condition (1A) in terms of logarithm, and differentiating with respect to time, yields:

(9A) $$-\rho + (1-\vartheta)(1-\sigma)\left[g(h) - g(c)\right] = g(\mu)$$

which, in consideration of (8A) and (6A), becomes:

(10A) $$-g(\mu) = \rho = -g(\lambda)$$

On substituting this value in (4A) and (5A), we obtain the optimal ratio between consumption and housing:

(11A) $$\left(\frac{c}{h}\right)^{*} = \frac{\sigma}{1-\sigma}\frac{(\rho - b_{11})(\rho - b_{22}) - b_{12}b_{21}}{(\rho - b_{11})} > 0 \quad \text{if}: \rho > b_{11} \text{ (sufficient condition)}$$

Rewriting constraints (1) and (2) in the text in terms of growth rates:

(1)’ $$g(k) = b_{11} + b_{12} \frac{h(t)}{k(t)}$$

(2)’ $$g(h) = b_{21} \frac{k(t)}{h(t)} + b_{22} - \frac{c(t)}{h(t)}$$

and equalizing them in consideration of condition (8A), bearing (11A) in mind, we obtain a second-order equation which defines the optimal ratio between housing and capital stock:

(12A) $$b_{12} (\frac{h}{k})^2 + \left[b_{11} - b_{22} + (\frac{c*}{h*}) \right] (\frac{h}{k}) - b_{21} = 0$$

which admits two roots, indicated as follows: $\left[\frac{h}{k}\right]_i^*$ con i=1,2 .

Finally, on substituting these values into equation (1)’, we obtain the optimal steady-state growth rate in function of the economy’s structural parameters (a, r, ρ, σ, γ) and of the policy parameters (β, τ_F ,τ_I , τ_h).

For significant values of the structural parameters, the root for i=2 generates a negatively and excessively high growth rate, which is not considered here. Then, it is sufficient to concentrate on the root for i=1. The steady state growth rate is described by equation (3) in the text.

References

Atkeson, A., Chari, V.V., Kehoe, P.J. (1999), *Taxing Capital Income: A Bad Idea,* Federal Reserve Bank of Minneapolis 23(3), 3-17.

Balducci, R. (2005), *Public Expenditure and Economic Growth. A Critical Extension of Barro's (1990) Model,* Economia Politica, n.2, agosto 2006, pp.163-172.

Barro, R.J. (1990), *Government spending in a simple model of endogenous growth,* Journal of Political Economy, 98, 103-25

Baxter, M., Jerman, U.J. (1999), *Household Production and the Excess Sensitivity of Consumption to Current Income,* American Economic Review 89(4), 902-920.
Berovec, J. , Fullerton, D. (1992), *A General Equilibrium model of Housing, Taxes, and Portfolio Choice,* Journal of Political Economy 100(2), 390-429.

Bye, B. , Avitsland, T. (2003), *The Welfare Effects of Housing Taxation in a Distorted Economy: A general Equilibrium Analysis,* Economic Modelling 20, 895-921.

Cannari L., D'Alessio G. (2006), La ricchezza degli italiani, Il Mulino, Bologna.

Coleman II, W.J. (2000), *Welfare and Optimum Dynamic Taxation of Consumption and Income,* Journal of Public Economics 76, 1-39.

Cremer, H. , Gahvari, F. (1998), *On Optimal taxation of Housing,* Journal of Urban Economics 43, 315-335.

Eerola, E. , Maattanen, N. ((2005), *The Optimal Tax Treatment of Housing Capital in the Neoclassical Growth Model,* Bank of Finland Research, D.P. n.10, 1-30

Englund, P. (2003), *Taxing residential Housing Capital,* Urban Studies, 40(5-6), 937-952.

Gavhari, F. (1985), *Taxation of Housing, Capital Accumulation, and Welfare: a Study in Dynamic Tax Reform,* Public Finance Quarterly 13(2), 132-160.

Gervais, M.(2002), *Housing Taxation and Capital Accumulation,* Journal of Monetary Economics 49(7), 1461-1489.

Gomme, P. ,Kydland, F.E., Rupert, P. (2001), *The Allocation of Capital and Time over the Business Cycle,* Journal of Political Economy 99(6), 1188-1214.

Guiso L., Jappelli T.,*Househols' Portfolio in Italy,* London, CEPR Discussion Paper, n.2549, 2000

Hendershott, P.H., White, M.(2000), *Taxing and Subsidizing Housing Investment: The Rise and Fall of Housing's Favored Status,* Journal of Housing Research 11(2), 257-Jones, L.E., Kurz K., Blossfeld H.P., *Home Ownership and Social Inequality in Comparative Perspective,* Stanford Universirty Press, 2004, California.

Manuelli, R.E., Rossi, P.E. (1997), *On the Optimal Taxation of Capital Income,* Jouranl of Economic Theory 73, 93-117.

Himmelberg C., Mayer C., Sinai T. (2005), *Assessing High House Prices: Bubbles, Fundamentals and Misperceptions,* Journal of Economic Perspectives, vol.19, n,.4, pp.67-92

McGrattan, E.R., Rogerson, R., Wright, R. (1997), *An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy,* International Economic Review 38(2), 267-290.

Poterba, J. (1992), *Taxation and Housing: Old Questions, New Answers,* American Economc Review 82(2),237-242.

Skinner, J. (1996), *The Dynamic Efficiency Cost of Not Taxing Housing,* Journal of Public Economics 59(3), 397-417.

Turnovsky, S.J., Okuyama, T. (1994), *Taxes, Housing and Capital Accumulation in a Two.Sector Growing Economy,* Journal of Public Economics 53, 245-267.