12. R&D models of economic growth and the long-term evolution of productivity and innovation

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12.1. INTRODUCTION

The ratio between the number of scientists and engineers engaged in research and development (R&D) and the level of total employment increased dramatically in the USA in the second half of the twentieth century. Let us call h_L the ratio between employment outside of R&D and total employment. In the USA $(1 - h_L)$ was nearly three times as large in 1993 as in 1950, with a pronounced upward fluctuation in the period 1960–70 due to government-funded R&D. Jones (2002) estimates that from 1950 to 1993 there was an even larger rise in the researchers/employment ratio in the set of G-5 countries (France, West Germany, Japan, the United Kingdom and the United States).

It is quite striking that the observed dramatic rise of R&D employment did not show up in the productivity figures. As is well known, the growth rate of GDP per hour tended to decline in the advanced countries after the 1950–70 'golden age'. The decline was less pronounced in the USA because this country did not enjoy the boom in productivity from technological catching up after the Second World War. Hence the US experience provides a more telling indication of the relation between R&D effort and productivity growth for a country located on the frontier of technological knowledge.[1]

I will refer the mentioned rise in the researchers/employment ratio as stylised fact (a) and the relatively constant growth rate of GDP per hour in the second half of the twentieth century as stylised fact (b).[2]

The question discussed in this chapter is how the R&D models developed within the recent revival of general-equilibrium-growth theory cope with the facts (a) and (b).[3] A similar question was addressed in an influential paper written by C.I. Jones and published in 1995. Jones observed how the R&D growth models developed to that date displayed a 'scale effect' of the number of researchers on the growth rate of GDP per capita. These models are criticised by Jones because the 'scale effect' is in striking contrast with the evidence. In the same paper he builds a model, which he defines as *semi-endogenous*, where innovations are still the outcome of purposeful and costly R&D effort, but the steady-state growth rate of output per capita is completely determined by the technological parameters and the rate of population growth. It is therefore independent of the level of population, of preferences, and of policy variables that do not affect technology. The family of R&D growth models with these properties is called here *non-endogenous*. By contrast, the endogenous R&D models of general-equilibrium growth are those where per-capita GDP growth depends upon preferences and/or policy variables generally. The basic structure of the endogenous and non-endogenous general-equilibrium models of economic growth is discussed in Sections 12.2, 12.3 and 12.4.

Partly as a reaction to Jones' critique, a second generation of endogenous R&D growth models appeared in the late 1990s. In this second generation, besides 'intensive' innovations that increase the productivity of the intermediate good produced in their sector of application, there are 'extensive' innovations, that increase the number of intermediate goods. In steady-state equilibrium, the number of intermediate goods (hence of sectors) grows at the population growth rate n, so that, in steady state, the number of intensive-researchers per sector is constant. This implies that the 'scale effect' on the rate of growth disappears. In other words, there is a dilution of the 'scale effect' across the growing number of intermediate-good sectors.

The steady-state predictions of the second-generation endogenous and also of the non-endogenous R&D growth models are still at odds with the evidence presented at the beginning of this introduction. The dramatic long-term rise of the R&D employment share $(1 - h_L)$ reveals that the long-term growth path of the US economy cannot find a theoretical approximation through the hypothesis that the economy has been growing in the neighbourhood of a single steady-state path.[4] The observed long-term rise of $(1 - h_L)$ and the approximately constant rate of productivity growth may be more consistent with the hypothesis of a transition path induced by exogenous parameter changes. This issue is addressed in Section 12.5.1. My conclusion here is that the changes in the technological parameters required to reconcile the stylised facts (a) and (b) above may be implausibly large.

In Section 12.5.2 I suggest that the failure of the R&D growth models to reconcile the constant productivity growth with the long-term rise of the R&D employment share can be interpreted as the result of technological assumptions that make abstraction from complementarity in production. Following a system-like view of technology which owes much to the contributions of Nathan Rosenberg, Joel Mokyr and many others and which can be traced back to Karl Marx, I stress the relation between the arrival of new technologies and the growth of variety and show how this may help to explain the stylised facts (a) and (b).

The focus of this chapter is on steady-state results. When transitions between different steady states are involved, e.g. in Section 12.5.1, the implicit assumption is that transitional dynamics are monotonic. Eicher and Turnovsky (1999b and 2001) show that non-monotonic transitional paths may exist in the non-endogenous growth models with two endogenously accumulating factors, knowledge A and capital K. In what follows the endogenously accumulating factors are capital K, intensive technical knowledge A and extensive technical knowledge N. A general analysis of the transition dynamics for the R&D growth models of this type is still lacking.[5] The discussion of how it may be relevant to the theme of this chapter is left to future work.

An important caveat must be added. In what follows, the rigid supply orientation of the general-equilibrium models of economic growth is taken for granted and is not questioned. This is not because the author is not aware of the biases that are introduced when co-ordination problems or stability in the disequilibrium dynamics are disregarded. These issues are simply outside the scope of this chapter.[6] In a similar vein, the chapter is unconcerned with the criticism that may be levelled at the use of capital aggregates in theoretical models. The attitude is simply to take the model predictions for what they are and discuss their consistency with broadly defined stylised facts.

12.2. A UNIFYING REPRESENTATION OF TECHNOLOGY

In what follows I build a framework which embeds different views of the relation between output growth and the generation of new inputs, as may be encountered in R&D growth models. This is done under a number of simplifying assumptions about technology that still enable us to discuss usually neglected issues, such as the role of complementarities and the relation between technological compatibility and knowledge spillovers. The main simplifying assumption is that the service characteristics of final output Y are unchanged throughout, that Y can be either consumed or accumulated in the form of capital and that it is produced by means of intermediate goods and labour. The number of available intermediate goods N_t changes through time as a result of innovation activities.

Assume that the number of service-characteristic types that exist in nature is finite. An intermediate good is a pair $(v, A_v) \in \Re^2_+$, where v is the intermediate–good variety (which identifies a class of functions performed by the good, that is, a composition of the associated flow of service characteristics) and A_v is the technological level, or generation, to which (v, A_v) belongs. In principle, we should expect A_v to have only an ordinal meaning, possibly with the further ordinal implication that later generations of a variety are also more productive. This is not, however, the interpretation we find in the new-growth literature, where A_v is an index leading to a cardinal productivity measure. The marginal product of (v, A_v) is a known time-invariant function of A_v (and possibly other variables). This leads to a time-invariant production possibility frontier, describing the productive potential of every possible present and future combination of intermediate goods.

12.2.1. Production of Material Goods

Final output Y is produced by means of intermediate goods and labour by perfectly competitive firms which, individually, face constant returns to scale. Following the R&D growth literature, we introduce a set of simplifying assumptions implying that at every date t only the highest (and latest) available technology level $A_{v\,t}$ of each variety v is used. This will be the case to the extent that the value of the productivity gain from using the latest generation of a given variety invariably dominates the cost differential associated with the same choice. [7] In fact, these models assume a particular substitutability relation between intermediate goods, to the effect that they enter the production function in an additively separable form. Recalling our simplifying assumptions, the individual production function is:

$$Y_t = N_t^{\gamma} L_{Y,t}^{1\text{-}\alpha} \left[\int_{v=0}^{N_t} A_{v,t}\ x_{v,t}^{\alpha} \partial v \right] \tag{12.1}$$

where x_v is a quantity of the intermediate-good variety v and L_Y is labour employment in the production of final output. It follows from (1) that the marginal product at t of the intermediate good (v, $A_{v,\,t}$) is $\alpha\ N_t^{\gamma} L_{Y,t}^{1\text{-}\alpha}\ A_{v,t}\ x_{v,t}^{\alpha-1}$. It is independent of the inputs of the other intermediate goods, although it may depend, if $\gamma \neq 0$, on the total *number* of intermediate goods cooperating with it. The above form of independence is interpreted here as resulting from the lack of technological complementarity between any two intermediate goods.

Intermediate goods are produced by local monopolists through a different set of activities. The reason why firms in the intermediate-good sector cannot be perfectly competitive is quite robust (Arrow, 1998; Romer, 1990b). The right to produce a new intermediate good involves an innovation cost that represents a fixed cost, because once the knowledge to produce a unit of a new good is acquired, it can be applied to the production of an indefinite number of units. If intermediate-goods production is otherwise subject to constant variable costs, we are faced with a clear case of increasing returns.

The input of the activity for producing one unit of (v, A_v) is a quantity of capital K which depends positively on the technology level A_v. K units of capital invested in the production of good (v, A_v) give rise to K/A_v^{ω} units of this good, where $\omega > 0$, thus implying that more capital intensive methods are required to produce intermediate goods of a later generation. Hence $K_v = x_v A_v^{\omega}$. Howitt (1999) adopts a similar increasing-capital-intensity assumption and claims that capital used in intermediate-goods production can be interpreted as human capital. The above specification implies that the average and marginal cost, in terms of final output, of producing (v, A_v) is rA_v^{ω}, where r is the rental price of capital. Since we abstract from depreciation, r is also the rate of interest.

The monopoly profit from producing $x_{v,t}$ is:

$$\pi_{v,t} = \alpha(1-\alpha)\, N_t^{\gamma} L_{Y,t}^{1-\alpha} A_{v,t} x_{v,t}^{\alpha}$$

Aghion and Howitt (1998, ch. 12) and Howitt (1999) obtain a monopoly output which is uniform across varieties and independent of A,[8] by setting $\omega = 1$. We hold to the latter simplifying assumption to obtain:

$$x_{v,t} = \alpha^{\frac{2}{1-\alpha}} N_t^{\frac{\gamma}{1-\alpha}} L_{Y,t} r_t^{\frac{1}{\alpha-1}} = x_t \tag{12.2}$$

In equilibrium, final output Y is then:

$$Y_t = N_t^{\gamma} L_{Y,t}^{1-\alpha} N_t A_t x_t^{\alpha} = \alpha^{\frac{2\alpha}{1-\alpha}} N_t^{\frac{1-\alpha+\gamma}{1-\alpha}} L_{Y,t} r_t^{\frac{1}{\alpha-1}} A_t \tag{12.3}$$

where A_t is the average technology level across intermediate goods:

$$A_t = \frac{1}{N_t}\left[\int_{v=0}^{N_t} A_{v,t}\,\partial v\right]$$

An equivalent equilibrium expression of Y_t is obtained by observing that, if h_K is the capital share employed in material, as opposed to knowledge, production, then in equilibrium we must have $h_{K,t} K_t / A_t = N_t x_t$. Hence:

$$Y_t = N_t^{\gamma} \left(h_{L,t} L_t\right)^{1-\alpha} N_t^{1-\alpha} A_t^{1-\alpha} (h_{K,t} K_t)^{\alpha} \tag{12.4}$$

It is then clear how the assumption $\gamma = \alpha - 1$ (see, for instance, Aghion and Howitt, 1998, ch. 12) sterilises the effects of the growing number of varieties on final output, which result from the additively separable way in which the single varieties enter the production function. Where these effects are not sterilised, because $(1 - \alpha + \gamma) > 0$, the production function corresponding to a constant technology level contains a form of increasing returns due to specialisation, as measured by N. The best known example along these lines is probably Romer (1990b), which assumes $\gamma = 0$.

Recalling that in steady state the rate of interest is constant, and the labour and capital shares employed in the (final and intermediate) output sector are also constant, equation (12.2) yields the steady-growth equation:

$$g_Y = g_L + \frac{1-\alpha+\gamma}{1-\alpha} g_N + g_A$$

where g_i is the proportional instant rate of change of variable i. In particular, if following Romer (1990b) we set the restrictions $\gamma = 0$ and $g_A = 0$, the above relation boils down to $g_Y = g_L + g_N$, where it is apparent that the growth rate of per-capita output is simply the growth rate in the number of specialised varieties.

12.2.2. Intensive Innovations

An intensive innovation in sector v arriving in the interval $t + \mathrm{d}t$ is the stochastic outcome of the innovation effort performed at t in this sector. The innovation contributes to shifting the technology frontier according to

$$\dot{A}_{t\,Max} = \frac{\delta}{N_t} A_{t\,Max} \tag{12.5}$$

and brings $A_{v,t}$ to the shifted frontier. Thus, access to the frontier technology level is available, but not costless, to every successful *intensive* innovator operating in sector v. The knowledge increment has elasticity +1 with respect to $A_{t\,Max}$ and elasticity –1 with respect to the number of sectors in the economy (Aghion and Howitt, 1998, ch. 12). The idea is here that the higher the number of sectors, the lower the impact of an innovation in sector v on the technology frontier.

The Poisson arrival rate of an intensive innovation in sector v at t is:

$$\phi_{v,t} = \lambda (u_{L,v,t} L_t)^{\theta} (u_{K,v,t} K_t)^{\xi} A_{t\,Max}^{\chi} \tag{12.6}$$

where $\xi \geq 0$, $\theta > 0$, λ is a constant, $u_{L,v}$ and $u_{K,v}$ are the fractions of total labour and capital invested in intensive R&D on variety v.

The returns offered by the investment of *rival*-resources in intensive R&D are constant or decreasing, depending on $\theta + \xi = 1$ (Barro and Sala-i-Martin, 1995, ch. 7), or $\theta + \xi < 1$. The second case arises if there is a congestion effect on the returns to R&D investment (Stokey, 1995; Howitt, 1999), with the result that the larger the rival resources invested in research, the higher the probability that independent innovation efforts produce the same outcome.

The parameter χ is meant to capture how the arrival rate is affected by the frontier knowledge stock $A_{t\ Max}$. There are two main forces at work here, which act in opposite directions. Thus, we may split the parameter χ into two components, $\chi = \chi_1 + \chi_2$. χ_1 is the so-called ‘complexity effect’: more advanced technology levels are progressively more difficult to discover as a result of the increasing complexity of the search activity. Thus, we have $\chi_1 < 0$. This is the assumption we find in a number of search models of R&D-based economic growth (Jovanovic and Rob, 1990; Stokey, 1995; Kortum, 1997).[9] The parameter $\chi_2 > 0$ captures the ‘standing on giants’ shoulders’ effect (see Merton, 1965; see also Caballero and Jaffe, 1993), which postulates that a higher frontier knowledge increases the probability of invention because an investment in intensive R&D creates the opportunity to exploit a knowledge spillover from the technology frontier to the innovators. This positive influence of knowledge on the innovation-success probability is distinct from and indeed adds to the influence of the stock of ideas on the size of the knowledge shift, which takes place *if* the innovation arrives (see (12.5) above). To this extent, it is unclear what are the grounds for assuming that the giants’ shoulders effect is positive and is close in absolute magnitude to the complexity effect. We shall see nevertheless that the restriction $\chi = \chi_1 + \chi_2 = 0$ (or other equivalent condition) is characteristic of the R&D endogenous-growth models.

The main simplifying hypothesis introduced with (12.6) is that the success probability of intensive R&D on variety v is independent of the distribution of the local stocks $A_{v,t}$. Together with (12.2) this implies that the intensive research effort and the arrival rate are uniform across sectors. Other formulations (see, for instance, Barro and Sala-i-Martin, 1995, ch. 7) relate the complexity effect and the giants’ shoulders effect for sector v to the local stock $A_{v,t}$. The same property of a uniform equilibrium arrival rate is however imposed also in this case, by means of ad hoc restrictions introduced to this end.

Since each agent engages in R&D independently of the agents in the same or in other sectors and the equilibrium research effort is uniform across sectors, the aggregate rate of intensive innovations is deterministic and equals:

$$N_t\phi_{v,t} = N_t\lambda(u_{L,v,t}L_t)^{\theta}(u_{K,v,t}K_t)^{\xi}A^{\chi}_{t\,Max} = N_t\lambda\left(u_L\frac{L_t}{N_t}\right)^{\theta}\left(u_{K,}\frac{K_t}{N_t}\right)^{\xi}A^{\chi}_{t\,Max} \quad (12.7)$$

where u_L and u_K are the aggregate labour and capital shares invested in intensive R&D.

12.2.3. Extensive Innovations

An 'extensive' innovation is the introduction of a new variety v. On the assumption that there is an external effect such that the technical knowledge in the economy affects the technology level of a new variety, a-not-too-implausible restriction is that the technology level distribution of a new variety corresponds to the technology level distribution across the existing varieties (Howitt, 1999). This implies that extensive innovations at t do not affect the average technology level in the economy A_t. An assumption to the same effect is that new varieties arriving at t have a deterministic technology level A_t (Peretto, 1998).

We assume that the extensive innovation effort is related to the creation of new varieties by the deterministic law:

$$\dot{N}_t = \beta(z_{L,t}L_t)^{\varepsilon}N_t^{\tau}(z_{K,t}K_t)^{\psi}A_t^{\nu} \equiv \phi_{N,t} \quad (12.8)$$

where β is a constant and z_L is the fraction of total labour employed in extensive R&D. We impose the restriction $\varepsilon > 0$, $\psi \geq 0$, $\tau \geq 0$, $\nu \geq 0$. The case $\varepsilon + \psi < 1$ indicates that there are decreasing returns with respect to the scale of the rival resources invested in extensive search. The restriction is referred to as the 'congestion hypothesis'. A positive τ bears the interpretation that a higher number of varieties amounts to a wider knowledge base in the economy as a whole and therefore facilitates the discovery of yet new varieties. If this is in itself quite plausible, far more questionable appear to be 'point restrictions' such as $\tau = 1$, or $\tau = 0$, as may be found, for instance, in the pure variety-extension model of Romer (1990b) and in Peretto (1998), respectively.

The parameter ν indicates how the production of an extensive innovation flow $\dot{N}$ of technology level A is related to the size of the average technology index A. If $\nu = 0$, then the cost (in terms of rival resources invested in extensive R&D) of producing a given innovation flow $\dot{N}$ with average technology level A is independent of A (Peretto, 1998). If $\nu > 0$ (< 0), this cost is decreasing (increasing) in A. The restriction $\nu > 0$ fits with the idea that the growth of technical knowledge along the quality dimension goes hand in hand with a growing 'complexity' of technology, which has a positive effect on the ease with which new varieties are discovered.[10]

12.3. STEADY-GROWTH EQUATIONS

A steady state, or balanced-growth path, is a particular constant-growth path such that the growth rate of every variable is constant *for ever*. Since the employment shares of the factors cannot exit the interval [0, 1], the definition immediately implies that the growth rate of such variables is zero on a balanced path.

The assumptions of Section 12.2.2 imply that the ratio $(A_{t\,Max} / A_t)$ converges to $(1 + \delta)$ (see Aghion and Howitt, 1998, p. 412). Assuming that convergence has already taken place, from (12.5) and (12.7) we obtain the following shift in the average technology level at time t, resulting from the intensive R&D in the N sectors:

$$\dot{A}_{t,Max} = \delta\lambda\left(u_{L,t}\frac{L_t}{N_t}\right)^{\theta}\left(u_{K,t}\frac{K_t}{N_t}\right)^{\xi} A_t^{\chi+1} \qquad (12.9)$$

Recalling that on a constant-growth path $\dot{A}_t$ and A_t grow at the same rate, using (12.4), (12.8) and (12.9) we write the steady-state growth equations:

$$g_A[-\chi] + (\xi + \theta)\, g_N - \xi\, g_K = \theta\, n \qquad (12.10)$$

$$-\nu\, g_A + (1 - \tau)\, g_N - \psi\, g_K = \varepsilon\, n \qquad (12.11)$$

$$-(1 - \alpha)\, g_A - (\gamma + 1 - \alpha)\, g_N + g_K\,(1 - \alpha) = (1 - \alpha)\, n \qquad (12.12)$$

If we define the variables $k \equiv K / N$, $l \equiv L / N$, so that

$$g_K = g_k + g_N,\ n = g_l + g_N,$$

then (10), (11) and (12) yield the following system:

$$\begin{bmatrix} -\chi & 0 & -\xi \\ -\nu & 1-\tau-\varepsilon-\psi & -\psi \\ -(1-\alpha) & -(\gamma+1-a) & 1-a \end{bmatrix}\begin{bmatrix} g_A \\ g_N \\ g_k \end{bmatrix} = \begin{bmatrix} \phi g_l \\ \varepsilon g_l \\ (1-a)g_l \end{bmatrix} \qquad (13)$$

12.3.1. Endogenous R&D Growth

Let $[I - \Gamma]$ be the square matrix in the left-hand-side of (13). If $[I - \Gamma]$ has a non-zero determinant, the steady-state growth rates of A, N and K are fully determined by equations (12.13), hence by technology, given the exogenous growth rate of population. Thus Det $[I - \Gamma] \neq 0$ states that preferences do not have any bearing on the speed of steady-state growth and policy measures by a government are equally ineffective, unless they are able to affect the technological parameters. It is then apparent that the crucial assumption of the endogenous R&D growth models is Det $[I - \Gamma] = 0$. In this case the coefficients in (12.13) are linearly dependent and additional equations are necessary to determine the steady-state growth rates of the variables. One missing equation is derived from the first-order conditions associated to the utility-maximisation problem:

$$Max: \int_{t=0}^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-(\rho - n)t} \partial t$$

subject to the flow budget constraint that per-capita consumption at tc_t is not negative and is constrained by wage and interest income minus the accumulation of stocks at t (see Barro and Sala-i-Martin, 1995, ch. 2). ρ is the rate of time preference and $(1/\sigma)$ is the constant inter-temporal elasticity of substitution. In particular, the proportional growth rate of c_t must satisfy:

$$g_c(t) = \frac{r_t - \rho}{\sigma}$$

where c is per capita consumption. In steady state $n + g_c = g_Y = g_K$.

The restriction Det $[I - \Gamma] = 0$ may be of course introduced in a number of ways. The standard practice of endogenous growth models with intensive R&D is to postulate the special case: $\chi = 0$ and $\xi = 0$ (see, for instance, Grossman and Helpman, 1991; Aghion and Howitt, 1992; Howitt, 1999; Peretto, 1998; Young, 1998; Barro and Sala-i-Martin, 1995; ch. 7). This is the case considered in the sequel of Section 12.3.1, yielding:

$$\frac{\dot{A}_t}{A_t} = \delta\lambda \left(u_{L,t} \frac{L_t}{N_t} \right)^{\theta} \tag{12.14}$$

As is also revealed by the first equation of system (12.13), with $\chi = \xi = 0$ consistency with steady state requires $g_N = n$, that is, $g_l = 0$. In particular, in the models where extensive innovations are not contemplated, so that N is constant, it is assumed that L is also constant and there is a scale effect of the intensive-research employment level on the growth rates of A and Y. This occurs in the pure quality expansion model of Grossman and Helpman (1991), Aghion and Howitt (1992) and Barro and Sala-i-Martin (1995, ch. 7). Jones (1995) draws attention to the lack of empirical corroboration for the hypothesis of a scale effect on the growth rate. In models with a growing population, equation (12.14) is reconciled with the lack of any scale effect on the steady-state rate of growth, by introducing special assumptions which make sure that L/N is constant (Howitt, 1999), or at least converges to a fixed steady-state value (Peretto, 1998; Young, 1998). With the simplified specification of equation (12.8) considered below ($\nu = 0$, $\psi = 0$), the required restriction is $\tau + \varepsilon = 1$. This implies:

$$\frac{\dot{N}_t}{N_t} = \beta z_{L,t}^{\varepsilon}\left(\frac{L_t}{N_t}\right)^{\varepsilon}$$

and using the steady-state condition $g_N = n$, this yields

$$mz_L = \left(\frac{n}{\beta}\right)^{\frac{1}{\varepsilon}} \qquad (12.15)$$

where m is the steady-state value of L/N. There are two different sets of steady-state solutions of the endogenous model, as specified above, which correspond to the possibility that: (1) the costs of one additional unit of labour effort invested in extensive or intensive R&D are identical; (2) these costs are not identical. Case (1) is considered in the next section, case (2) in appendix A.

We shall proceed under the further simplifying assumption $\gamma = \alpha - 1$ (see equation 12.12), so that $g_K = g_A + n$. Thus: $g_c = g_A = (r - \rho)/\sigma$.

Suppose the only cost of one additional unit of labour effort in extensive or intensive research is the forgone opportunity of obtaining the wage rate w by selling that unit in the labour market. Free entry in research implies that, if the equilibrium levels of intensive and extensive R&D are positive, then the private instantaneous marginal returns from innovation effort must be identical between the two activities and must be equal to the wage rate w.

$$\left(\frac{\phi_{v.t}}{u_{L,v,t}L_t}\right)v_{v,t} = \lambda\left(u_{L,t}\,\frac{L_t}{N_t}\right)^{\theta-1}V_t = w_t = \left(\frac{\phi_{N,t}}{z_{L,t}L_t}\right)V_{N,t} = \beta\left(z_{L,t}\,\frac{L_t}{N_t}\right)^{\varepsilon-1}V_{N,t} \qquad (12.16)$$

where $V_{v,t} = V_t$ is the expected value of a quality innovation in any sector v at time t, and $V_{N,t}$ is the expected value of an extensive innovation at time t. Moreover, with our production function (12.4) we have:

$$w = (1-\alpha)h_L^{-\alpha}q^{\alpha}A \tag{12.17}$$

where $q \equiv K/AL$.

Let $v_t \equiv V_t/\mathrm{A}_{t,Max}$ and $v_{N,t} \equiv V_{N,t}/A_t$; in other words, v_t and $v_{N,t}$ are the productivity adjusted values at time t of an intensive and extensive innovation, respectively. From (12.16) and (12.17):

$$v_t = \frac{1-\alpha}{\lambda(1+\delta)} h_L^{-\alpha} q^{\alpha} \left(u_{L,t} \frac{L_t}{N_t} \right)^{1-\theta} \tag{12.18}$$

$$v_{N,t} = \frac{1-\alpha}{\beta} h_L^{-\alpha} q^{\alpha} \left(z_{L,t} \frac{L_t}{N_t} \right)^{1-\varepsilon} \tag{12.19}$$

Moreover, one obtains the asset equations (see Aghion and Howitt, 1998, pp. 109–10):

$$\dot{v}_t = [r_t + \phi_t]\, v_t - \pi_t \tag{12.20}$$

$$\dot{v}_{N,t} = [r_t + \phi_t]\, v_{N,t} - \pi_t \tag{12.21}$$

where π_t is the productivity adjusted profit of a local monopolist. It is worth recalling that, since an extensive innovation will be displaced by an intensive innovation in the same sector, the expected obsolescence rate takes the same value ϕ_t for extensive *and* intensive innovations.

Since the productivity adjusted value of extensive and intensive innovations are identical in equilibrium, $v_t = v_{N,t}$, which in steady state can be written:

$$(1+\delta)\lambda u_L^{\theta-1} m^{\theta} = \beta z_L^{\varepsilon-1} m^{\varepsilon} \tag{12.22}$$

Using (12.14) and (12.15) we obtain:

$$\frac{u_L}{z_L} = \left[(1+\delta)\lambda n^{\frac{\theta-\varepsilon}{\varepsilon}} \beta^{\frac{-\theta}{\varepsilon}} \right]^{\frac{1}{1-\theta}} \tag{12.23}$$

$$g_A = \delta\left[\lambda(1+\delta)^{\theta} n^{1-\varepsilon} \beta^{\frac{-\theta}{\varepsilon}}\right]^{\frac{1}{1-\theta}} \tag{12.24}$$

In the special, but convenient case $\theta = \varepsilon$ (12.23) and (12.24) simplify to:

$$\frac{u_L}{z_L} = \left[(1+\delta)\lambda\beta^{-1}\right]^{\frac{1}{1-\theta}} \tag{12.23′}$$

$$g_A = \delta n\left[\lambda(1+\delta)^{\theta} \beta^{-1}\right]^{\frac{1}{1-\theta}} \tag{12.24′}$$

Thus we reach the striking conclusion that in the endogenous model as specified above, an identical marginal innovation cost for intensive and extensive R&D makes (u_L/z_L) and g_A depend only on technological parameters. Instead, the steady-state shares u_L, z_L, and h_L depend also on the preference parameters ρ and σ. (See note 13, which refers to the special case $\theta = \varepsilon$.)

The reason why the model still qualifies as endogenous is that a policy variable such as an innovation subsidy (see Aghion and Howitt, 1998, p. 419) would affect the rate of growth if it exerted an asymmetric influence on the cost from one additional unit of labour effort in extensive and intensive R&D. For a discussion of this point, the reader is referred to the case considered in Appendix A, where the cost asymmetry does not arise from a policy variable, but from a slight generalisation of the innovation technology considered above.

12.3.2. Non-endogenous R&D Growth

Referring back again to system (12.13), the crucial assumption of the non-endogenous R&D growth models is Det$[I - \Gamma] \neq 0$. In particular, referring to the case $[I - \Gamma]^{-1} > 0$, standard theorems of linear algebra lead to the following proposition, which shows that the result similar in spirit to be found in Eicher and Turnovsky (1999a) extends to our economy with expanding varieties and technology levels.

Proposition 1: Assume $\Gamma \geq 0$. Assume also that, for each row, the row sum of the elements of Γ is positive and lower than 1. Then, for every $n > 0$, there exist positive values g_A, g_N, g_K that are solutions to (12.10), (12.11) and (12.12) and such that $g_l = n - g_N > 0$.
Recalling that $0 < \alpha < 1$, a quick look at equation (12) reveals:

Proposition 2: If, in addition to the assumptions of proposition 1, we have $(\gamma + 1 - \alpha) \geq 0$, then $g_K > n$ (positive per-capita-output growth).

Remark 1: The *if* condition of Proposition 2 amounts to the existence of increasing returns to scale in the output sector. The assumption of Proposition 1 implies, but is not equivalent to, aggregate decreasing returns to scale in extensive and intensive search.

Thus, where the equations of system (12.13) are not linearly dependent (notably, a condition of full measure in the relevant parameter space) the steady-state growth rates of output, technology levels and varieties are completely determined by population growth and the technological parameters. These rates are therefore independent of preferences and of savings rates in particular.

The above propositions extend to a three-sector environment the formal characterisation of the class of two-sector non-endogenous growth models first laid down by Eicher and Turnovsky (1999a). From a formal viewpoint the seminal paper of Arrow (1962), where technology accumulation is driven by learning rather than deliberate R&D investment, belongs to the same class. Within the family of R&D growth models, the best-known non-endogenous example is probably that provided by Jones (1995; see also Jones, 1998a and 2002), where the author abstracts from the expansion of varieties, so that $g_N = 0$ and $g_l = n > 0$. In particular, Jones (1995) assumes $\xi = 0$ (no physical capital input in R&D) and $0 < -\chi < 1$, so that his two-sector version of system (12.13) boils down to

$$\begin{bmatrix} -\chi & 0 \\ -(1-\alpha) & (1-\alpha) \end{bmatrix} \begin{bmatrix} g_A \\ g_K \end{bmatrix} = \begin{bmatrix} \theta n \\ (1-\alpha)n \end{bmatrix}$$

and the conditions of propositions 12.3.2.1 and 12.3.2.2 are trivially satisfied.

Interestingly, the steady-state relation $g_c = g_A = (r - \rho) / \sigma$ continues to hold, but the direction of causality at work here is such that, given n, technology determines g_A and r is then determined by g_A and preferences. As is discussed in Appendix A, in the endogenous model with asymmetric cost of innovation effort between extensive and intensive R&D, technology and preferences simultaneously determine g_A and r.

12.4. IS n AN UPPER BOUND FOR g_N?

As it turns out, the available examples of endogenous and non-endogenous R&D growth models share the prediction that, *in steady state*, the expansion of varieties proceeds at a pace which is *not faster* than the pace of population growth. In particular, $g_N = n$ in the endogenous and $g_N < n$ in the non-endogenous models considered above. On closer examination, however, these predictions are the by-product of quite special assumptions. Both the endogenous and the non-endogenous model admit extensions such that g_N may be greater than n. The point is considered in Appendix B.

12.5. RESEARCH EMPLOYMENT AND PRODUCTIVITY

A second and deeper problem is posed to the R&D growth models by the stylised facts (a) and (b) mentioned in the introduction. These stylised facts are at variance with the possibility of approximating (if at a very aggregate level) the long-term evolution of innovation activity and productivity growth in the US through the hypothesis that this economy has been growing in the neighbourhood of a single steady-state path. More specifically, endogenous and non-endogenous models alike are faced with the problem of:

1. explaining how the rising researchers/employment ratio $(1 - h_L)$ can be reconciled with the behaviour of productivity growth;
2. identifying the causes of the rising researchers/employment ratio.

A first way of answering these questions is to suppose that the rise in $(1 - h_L)$ corresponds to a transitions path with constant growth rate g_A induced by *exogenous* changes in one or more technological parameters.

A second and more ambitious way is much in the spirit of Pasinetti (1981) and searches for *rules of structural change* that may get closer to explaining the observed phenomena without resorting to exogenous parameter changes. In the remainder of this chapter we shall expand on these two lines of investigation.

To this end, I shall refer to the simplified versions of system (12.13) that feature in 'standard examples' of endogenous and non-endogenous R&D growth models. In particular, physical capital is not an input to innovation activity, intensive and extensive, hence $\xi = 0$, $\psi = 0$; the productivity of the extensive innovation effort does not depend on the technology level A, that is, $\nu = 0$; the aggregate production function does not depend on the number of varieties N, thus $\gamma = \alpha - 1$. In addition, I introduce the simplifying restriction $\varepsilon = \theta$, that is, the elasticity of innovation output with respect to R&D labour effort is uniform across extensive and intensive innovations.

12.5.1. Looking for Appropriate Parameter Changes

Referring to the US experience in the second half of the twentieth century, we may observe how the rate of interest and the capital output ratio have been 'relatively constant'[11] over the period. Since the model structure implies $\sigma g_A + \rho = r = \alpha^2 K / Y$, using stylised fact (b) we derive the restriction that α has been constant; in this Section I am also led to formulate the 'working hypothesis' that the preference parameters σ and ρ were unchanged throughout. With this situation in mind I consider what, if any, changes of the technological parameters of the non endogenous and endogenous models can answer the issues posed under (1) and (2) above.

With the assumptions of proposition 1 in place, in particular $0 < -\chi < 1$, $\varepsilon + \tau < 1$, the non-endogenous model yields the steady-state predictions:

$$g_Y = g_A + n$$

$$g_N = \frac{\varepsilon n}{1-\tau}$$

$$g_A = \frac{\theta(1-\tau-\varepsilon)n}{-\chi(1-\tau)}$$

The growth rate of per capita output is independent of δ, the proportional productivity effect of quality innovations; it is also independent of λ and β, the parameters that, for any given innovation effort, regulate the arrival rates of intensive and extensive innovations, respectively.

Since the (expected) productivity-adjusted values v_t, $v_{N,t}$ of intensive and extensive innovations are identical, free entry in R&D implies the following equilibrium condition at every date t:

$$\frac{u_{L,t}}{z_{L,t}} = \left[A^{\chi}_{t,Max}(1+\delta)\frac{\lambda}{\beta} N_t^{1-\tau-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{12.25}$$

On every equilibrium path sustained by smooth changes of λ, β and δ:

$$g_{A,t} = \left[\frac{\dot{\delta}_t}{1+\delta_t} + \frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{\beta}_t}{\beta_t} + (\theta-1)\left(\frac{\dot{u}_{L,t}}{u_{L,t}} - \frac{\dot{z}_{L,t}}{z_{L,t}} \right) + (1-\tau-\varepsilon) g_{N,t} \right] \frac{1}{-\chi} \tag{12.26}$$

On a growth path with constant growth rates of A and N:

$$g_{N,t}\left[\frac{\dot{\beta}_t}{\beta_t}+\theta\left(\frac{\dot{z}_{L,t}}{z_{L,t}}+n\right)\right](1-\tau)^{-1} \tag{12.27}$$

$$g_{A,t}=\left[\frac{\dot{\delta}_t}{\delta_t}+\frac{\dot{\lambda}_t}{\lambda_t}+\theta\left(\frac{\dot{u}_{L,t}}{u_{L,t}}-\frac{\theta}{1-\tau}\frac{\dot{z}_{L,t}}{z_{L,t}}-\frac{1}{1-\tau}\frac{\dot{\beta}_t}{\beta_t}+\frac{1-\tau-\varepsilon}{1-\tau}n\right)\right]\frac{1}{-\chi}$$

Substituting from (12.27) into (12.26) we obtain:

$$-\frac{\dot{\delta}_t}{\delta(1+\delta_t)}=\frac{\dot{u}_{L,t}}{u_{L,t}}-\frac{\dot{z}_{L,t}}{z_{L,t}} \tag{12.28}$$

Using (12.28), (12.27) and (12.25), a transition path with

$$\frac{\dot{\delta}_t}{\delta_t}\neq 0,\ \frac{\dot{\lambda}_t}{\lambda_t}\neq 0,\ \frac{\dot{\beta}_t}{\beta_t}\neq 0$$

and constant growth rates $g_{A,t}$, $g_{N,t}$ satisfies:

$$\text{if } \frac{\dot{\delta}_t}{\delta_t}<0\text{, then } \frac{\dot{\delta}_t}{\delta_t}+\frac{\dot{\lambda}_t}{\lambda_t}>\frac{\dot{\beta}_t}{\beta_t};\quad \text{if } \frac{\dot{\delta}_t}{\delta_t}>0,\ \text{then } \frac{\dot{\delta}_t}{\delta_t}+\frac{\dot{\lambda}_t}{\lambda_t}<\frac{\dot{\beta}_t}{\beta_t}.$$

Moreover, the steady state share $(u_L + z_L)$ is independent of λ and β and satisfies[12] $\partial(u_L + z_L)/\partial\delta < 0$, if σ is not too lower than 1. The above considerations suggest the conjecture that a transition path with rising share $(u_L + z_L)$ and constant growth rate of productivity is explained by

$$\frac{\dot{\delta}_t}{\delta_t}<0 \text{ and } \frac{\dot{\delta}_t}{\delta_t}+\frac{\dot{\lambda}_t}{\lambda_t}>\frac{\dot{\beta}_t}{\beta_t}.$$

To gain some understanding of the problems posed by this line of reasoning, it is worth observing that the dramatic rise of $(u_L + z_L)$ would be obtained through partly offsetting changes of u_L and z_L. Recalling (12.28), our conclusion here is that the rates of change of the technological parameters δ and λ which are required to explain the stylised facts (a) and (b) may be implausibly high.

In addition to the simplifying assumptions stated at the outset of Section 12.5, the endogenous model we are considering assumes $\chi = 0$, $\varepsilon + \tau = 1$.

The innovation technology is that considered in Section 12.3.1 generating a symmetric cost from one additional unit of labour effort across extensive and intensive innovations.[13] Following the same line of reasoning explained above, we obtain that a transition path with smooth changes of λ, β and δ and constant growth rates $g_{A,t}$, $g_{N,t}$ satisfies (12.28) and

$$\frac{\dot{\delta}_t}{\delta_t}+\frac{\dot{\lambda}_t}{\lambda_t}=\frac{\dot{\beta}_t}{\beta_t}+\delta_t\left(\frac{\varepsilon}{\delta_t(1+\delta_t)}\right).$$

The difficulties encountered by the line of reasoning under investigation are therefore similar to those discussed for the non-endogenous model.

12.5.2. Growth and Structural Change

In a recent paper, Kongsamut et al. (2001) suggest that the long-term rise in the service-employment share has to do with changes in the composition of consumers' expenditure associated with the long-term rise of per-capita income. A tradition in economic theory, from Kuznets (1957) to Pasinetti (1981) had already emphasized this order of phenomena. In a similar vein, I introduce in this section the hypothesis that the long-term rise of the research employment share may be explained by a slow, almost negligible secular rise of the intertemporal elasticity of substitution associated to the long-term rise of GDP per capita.[14] In what follows, the focus of my analysis is not that of giving a detailed specification of the hypothesis, but is that of suggesting a line of argument explaining how stylised facts (a) and (b) may be reconciled.

The explanation rests upon the complementarity between the goods and methods used in production. However often neglected, the idea is far from new. Perceptive remarks on the relevance of this notion can be found in Marx's volume I of *Capital*. In chapter XV it is emphasised that the successful exploitation of new engineering and scientific principles in production required the emancipation of technology from the pre-existing set of tools[15] and that 'a radical change in the mode of production in one sphere of industry involves a similar change in other spheres' (Marx, 1954, p. 362). In the new-growth literature, the problem of complementarity between intermediate goods has been introduced in relation to the idea of a sequence of general-purpose technologies (GPTs). The adoption of a GPT requires the previous creation of a set of intermediate goods that are specific to it.[16]

I suggest that a similar set of ideas can be conducive to phenomena of structural change within a framework which is borrowed, with some important variations or qualifications, from the R&D growth models considered in this chapter.

For the sake of simplicity, let us assume away the problem of extensive R&D by assuming that at every date there is an unchanging continuum of intermediate-good varieties ordered on $\Re_+$. To employ these varieties in production, their appropriate technology level must be developed. $[0, \Lambda_A]$ is the set of complementary intermediate-good inputs *necessary* to implement the technology level A in the production of final output. N_t is the number of intermediate goods *used* at t. There is only one final good Y. Its production function is:

$$Y_t = N_t^{\alpha-1} L_{Y,t}^{1-\alpha} \left[\int_{v=0}^{\infty} P(A_{v,t}) x_{v,t}^{\alpha} \partial v \right]$$

where $P(A_{v,t})$ is the productivity index associated to the technology level $A_{v,t}$ of variety v. with $P(A_{v,t}) = A$, if $0 \leq v \leq \Lambda_A$ and $A_{v,t} = A_{j,t} = A$ for all v, $j \in [0, \Lambda_A]$; $P(A_{v,t}) = 0$ otherwise. This assumption formalises a strong form of incompatibility between intermediate goods of a different technology level. We say that technology level A has been implemented if $A_{v,t} = A_{j,t} = A$ for all $v, j \in [0, \Lambda_A]$ .Variety v is *necessary* to the implementation of A if and only if $\in [0, \Lambda_A]$.

If technology level $A(t)$ is implemented at time t, there is an instantaneous knowledge spillover such that $A_{v,t} = A(t)$ for every $v \in [0, \infty]$. The implementation of a *higher* technology level is instead costly, because it requires the higher level is independently developed for every necessary variety as the result of a deliberate R&D effort. The number $\phi_{v,t}$ of intensive innovations in sector v at t evolves according to the *deterministic* process:

$$\phi_{v,t} = \lambda\,(u_{L,v,t}\,L_t)^{\theta}\,A_{v,t}^{\ \chi}$$

If every innovation has a proportional effect δ on the technology level $A_{v,t}$, we obtain:

$$\dot{A}_{v,t} = \delta\lambda \left(u_{L,t} \frac{L_t}{N_t} \right)^{\theta} A_{v,t}^{\chi+1} \tag{12.29}$$

Higher technology levels are of higher complexity and their implementation requires a larger number of necessary intermediate inputs. Assume that the number of necessary varieties evolves according to:

$$\Lambda_{A(t)} = A_t^{\eta} \qquad \eta > 0$$

This implies that, if $g_{\Lambda(t)}$ is the proportional growth rate of $\Lambda_{A(t)}$, then:

$$g_{\Lambda(t)} = \eta\, g_{A(t)} \tag{12.30}$$

The strong complementarities of the form described above imply that the market implementation of a higher technology level will face a host of co-ordination problems. Here we are not concerned with this feature, however important it may be. Our aim is simply to show that equilibrium paths on which the productivity index A_t grows at a positive constant rate $g_A > 0$ are *not* steady states and have a rising share u_L of R&D employment.

In equilibrium, $N_t = \Lambda_{A(t)}$. With g_A constant, from (12.29) and (12.30) we obtain:

$$-\chi g_A = \theta\left(n + \frac{\dot{u}_{L,t}}{u_{L,t}} - g_\Lambda(t)\right)$$

$$\left(\eta - \frac{\chi}{\theta}\right)g_A = \left(n + \frac{\dot{u}_{L,t}}{u_{L,t}}\right)$$

Recalling that the 'congestion effect' in R&D implies $\theta < 1$, and that our considerations suggest $\chi < 0$, it is easy to see that, given n, the higher the value of η, the higher the growth rate $\dot{u}_{L,t}/u_{L,t}$ required to elicit a given productivity growth g_A. Thus, with η sufficiently large, the value $g_A \approx 0.02$ prevailing in the period 1950–93 would not have been possible in the presence of a constant labour share in R&D. Indeed, a growth rate g_A of the observed dimension cannot be a steady-state growth rate and cannot be sustained 'for ever'.

If the argument above offers a tentative explanation of how the long-term rise of the researchers/employment ratio can be reconciled with a constant growth rate of productivity, what is yet to be explained is the source of the rising researchers/employment ratio.

Here I suggest, as a working hypothesis to be explored by future work, that the preference structure with constant inter-temporal elasticity of substitution is replaced by a preference structure such that the rising per-capita consumption causes a slowly rising inter-temporal elasticity of substitution. Since h_L is close to 1 and u_L is close to zero,[17] the required change in σ does not have to be large, since a very small, seemingly negligible, shift away from employment in manufacturing in favour of research is sufficient to explain that:

(1) $\dot{h}_{L,t}/h_{L,t}$ is negative but very close to zero, as in the data;

(2) $\dot{u}_{L,t}/u_{L,t}$ is positive and significantly large, as in the data.

12.6. CONCLUSIONS

In spite of the statements to the contrary (Jones, 1995; Aghion and Howitt, 1998, ch. 12), growth models that avoid the scale effect of R&D employment on productivity growth do not explain the evidence on R&D employment and productivity growth in the US. Indeed, the stylised facts (a) and (b) mentioned in the introduction are not easily reconciled within the standard steady-state hypothesis.

The first reason offered in this chapter is that cross-sector research spillovers are less extensive than is normally assumed in R&D models: After a new basic idea is first discovered, the development and profitable implementation of the same idea in the production of final utput is a costly process. A second reason is that the number of complementary inputs necessary to implementa technology level A in the production of final output is likely to be an increasing function of A. The further assumption of complementarities in the form of strong incompatibilities between intermediate goods of a different technology level yields the result that structural change in the form of a rising R&D employment share is a necessary condition for the sustained growth rate of productivity experienced in the US during the second half of the twentieth century.

APPENDIX A: ASYMMETRIC INNOVATION COST

Suppose that every unit of labour invested in R&D at time t is combined with a quantity of capital $A_{t,Max}\ T_A$, in the case of intensive R&D and $A_t\ T_N$ in the case of extensive R&D. In this section I assume $T_N \neq T_A$. In other words, labour and capital are perfectly complementary inputs to intensive and extensive innovation activities, but the ratio between the two inputs is different in the two sets of activities, even after adjustment is made for the productivity levels $A_{t,Max}$ and A_t. The case $T_N = T_A$ yields conditions identical to those obtained in Section 12.3.1, with the understanding that terms K and q must be replaced everywhere with $h_K K$ and $h_K q$, where h_K is the fraction of total capital employed in the output sector (to produce intermediate goods). u_K and z_K are the fractions of total capital employed in intensive and extensive R&D, respectively. With this notation, and assuming for simplicity $\theta = \varepsilon$, the procedure followed in Section 12.3.1 yields:

$$\frac{u_L}{z_L} = \left\{ \frac{\lambda(1+\delta)\left[(1-\alpha)+\alpha^2\left(\frac{r}{\alpha^2}\right)^{\frac{1}{1-\alpha}} T_N\right]}{\beta\left[(1-\alpha)+\alpha^2\left(\frac{r}{\alpha^2}\right)^{\frac{1}{1-\alpha}} T_A\right]} \right\}^{\frac{1}{1-\theta}}$$

Hence u_L/z_L is related to the steady-state rate of interest, which depends on the preference parameters ρ and σ. In particular, it can be easily checked that the sign of $\partial(u_L/z_L)/\partial r$ is positive if $T_N - T_A > 0$ and is negative if $T_N - T_A < 0$. Moreover, similar considerations apply to the relation between g_A and the rate of interest. We can write:

$$g_A = \frac{r-\rho}{\sigma} = f(r,\lambda,\delta,\beta,\alpha,\theta,T_N,T_A)$$

If $T_N - T_A \neq 0$, then r is a non-redundant argument of the function $f(\,)$ and, given n, g_A and r are simultaneously determined by technology and preferences. If $T_N = T_A$ the simultaneity collapses and g_A is determined by (12.24′).

APPENDIX B: EXISTENCE OF SOLUTIONS WITH $g_N > n$

It is enough to provide two examples: one for the endogenous model and one for the non-endogenous model. In both examples the simplifying restriction $\gamma = \alpha - 1$ holds so that $\mathrm{Det}[I - \Gamma] = -(\xi + \chi)(1 - \tau - \varepsilon - \psi)(1 - \alpha)$.

For the endogenous model with $\chi = 0$, $\xi > 0$, $\upsilon > 0$, the crucial restriction $\mathrm{Det}[I - \Gamma] = 0$ is fulfilled by $\tau + \varepsilon + \psi = 1$. In this case

$$\dot{N} = N\beta z_L^{\varepsilon}(L/N)^{\varepsilon} A^{\upsilon}$$

which in steady state requires $\varepsilon(n - g_N) + \upsilon g_A = 0$. If $0 < \upsilon < \varepsilon$, this yields $g_{Ki} = g_A + n > g_N$. Since from (12.10) $g_K = g_N - (\theta/\xi)(n - g_N)$ we have that $g_N > n$ and $g_A > 0$ are consistent with a steady state path.

For the non-endogenous model it is sufficient to assume $\tau < 1$, $\tau + \varepsilon + \psi > 1$; $\xi + \chi > 0$, ν and ψ sufficiently close to zero.

NOTES

1. There are instances of R&D activities performed in a given country which exert their productivity effects mainly outside the country: think of a new treatment for curing a tropical disease discovered in the USA or in Germany. The view taken in this chapter is that this type of phenomenon is far from explaining the qualitative evidence presented in the text. I thank Francesco Pigliaru for drawing my attention to this point.
2. To reconcile facts (a) and (b), two candidates come to mind. (1) There has been a fall in the average effect of innovations on measured productivity. This may be at least partly due to the fact that official statistics underrate the qualitative changes in goods and the improvement in their service characteristics (Nordhaus, 1997). Alternatively, or in addition to the previous cause, it may be increasingly difficult to produce the same proportional improvement in the service characteristics of goods. Hence, the productivity gain tends to fall in the more recent innovations. Robert Gordon (2000) compares the effects on well-being of the 'new economy' to those produced by the great innovations during the second industrial revolution. He concludes that the effects of the former do not bear comparison with those of the latter. (2) A different, but compatible, line of explanation is a fall in the average productivity of R&D labour, as measured by the number of innovations per unit of research effort. A fall of this kind has certainly taken place, if the number of innovations is measured through the number of patents, granted or applied for (Griliches, 1989, 1990). Measures of this type are strongly biased not only by changes in the 'productive capacity' of institutional patent agencies (e.g. the US Patent Office), but also by changes in the propensity to apply for a patent. Microeconomic studies (Lanjow and Schankerman, 1999) indicate that a lower fall of the productivity of R&D labour is obtained if the aggregate innovation output is obtained by weighting patents by means of indicators of their technological and economic importance. This is related to point (1) above.
3. We shall not consider other families of models where growth is likewise driven by innovations, let alone the huge microeconomic literature on R&D.
4. By definition, on a steady-state path the growth rate of every variable is constant for ever. Since the employment shares are bounded between zero and one, their unique admissible steady-state growth rate is zero.
5. Peretto (1998) reports on the transition dynamics of an R&D growth model where the endogenously accumulating factors are only A and N. In the transition dynamics results of Aghion and Howitt (1998, pp. 109–15), the endogenous factors are A and K.
6. Still, in reading it, it is best to bear in mind what is implied by the seminal work by Jacob Schmookler (1966) on innovation and growth: the interest in the causes of the long-term growth of GDP per capita, as distinguished from the GDP level, is at best only a partial justification for the rigid supply orientation of general-equilibrium growth models.
7. The assumption is not fully realistic. Even granting that A_v amounts to a productivity index, we should in general expect the flow of service characteristics associated with (v, A_v) to depend upon the type and quantity of other intermediate goods with which (v, A_v) co-operates within a production activity. If there are strong complementarities between different intermediate goods, the best-practice technology level of variety v at t may not be the highest available. Compatibility constraints may in fact imply that it is inefficient to use very different technology levels of complementary varieties in the same activity. Complementarities of this sort are simply ruled out in most (an exception is Helpman and Trajtenberg (1994); see section 12.5.2 below) R&D growth models.
8. If $1 > \omega$, then the monopoly output is positively related to the technological advance $A_{v,t}$.
9. Realistic as it may be, the positive correlation between the technology-frontier index and the search difficulty must be simply assumed and cannot find a micro foundation within a

formal framework which does not lend itself to consider the feedback of innovations on the complexity of the search space.

10. As before, since the present framework cancels from view the rising complexity of the technology space, the treatment of this feature can be at best evocative.
11. At least in the sense specified in the introduction to this chapter.
12. $(u_L + z_L)^{-1} = 1 + \dfrac{\theta n(1-\tau-\varepsilon)(1+\sigma\delta) - \chi\delta[(\rho-n)(1-\tau-\varepsilon)+\rho\varepsilon]}{(1+\delta)(1-\tau-\varepsilon)\alpha\theta n - \chi\delta\varepsilon\alpha n}$
13. The research employment shares are:

$$u_L = \frac{\left[(1+\delta)\dfrac{\lambda}{\beta}\right]^{\frac{1}{1-\varepsilon}}}{1+\dfrac{\rho}{\alpha n}\left[1+\dfrac{\sigma\delta+1}{(1+\delta)\alpha}\right]\left[(1+\delta)\dfrac{\lambda}{\beta}\right]^{\frac{1}{1-\varepsilon}}}$$

$$z_L = \left[1+\frac{\rho}{\alpha n}\left[1+\frac{\sigma\delta+1}{(1+\delta)\alpha}\right]\left[(1+\delta)\frac{\lambda}{\beta}\right]^{\frac{1}{1-\varepsilon}}\right]^{-1}$$

It can be easily verified that: $\partial u_L/\partial(\lambda/\beta)>0$; $\partial z_L/\partial(\lambda/\beta)<0$; $\partial(u_L+z_L)/\partial(\lambda/\beta)<0$ and $\partial(u_L+z_L)/\partial\delta<0$ if $\sigma>[(1+\delta)\rho-n]/\delta n$; $\partial(u_L+z_L)/\partial(\lambda/\beta)>0$ and $\partial(u_L+z_L)/\partial\delta > 0$ if $\sigma\leq 1$.

14. The hypothesis implicitly assumes some measurement error leading to a (very) mild under-evaluation of productivity growth. See above, note 2.
15. ‘It is only after considerable development of the science of mechanics, and accumulated practical experience, that the form of a machine becomes settled entirely according to mechanical principles, and emancipated from the traditional form of the tool that gave rise to it’ (Marx, 1887, p. 362, n. 1).
16. When the GPT *s* first appears a labour share is shifted from manufacturing to R&D (phase 1); next, after the intermediate goods required by *s* have been invented all employment is shifted to manufacturing until the GPT (*s* + 1) arrives (phase 2). The idea is exploited by Helpman and Trajtenberg (1994) and Aghion and Howitt (1998) to study the relation between growth and cycles. The notion of a steady state is correspondingly extended by these authors to the effect that in an economy with a constant population ‘a steady-state equilibrium is one in which people choose to do the same amount of research each time the economy is in phase 1 …’ (Aghion and Howitt, 1998, p. 248).
17. The US researchers/employment ratio was 0.008 in 1993 (see Jones, 2000, p. 16).